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AN ELEMENTARY LOGIC


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ELEMENTARY LOGIC

BY

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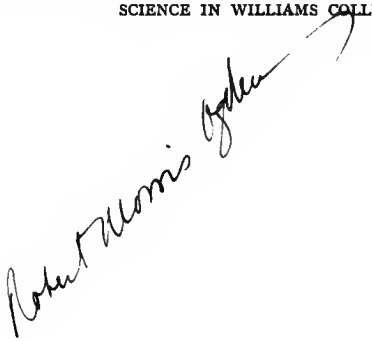

New York

THE MACMILLAN COMPANY

LONDON: MACMILLAN & CO., LTD.

1906

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Set up and electrotyped. Published April, 1906.

Norwood Press
J. S. Cushing & Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

PREFACE

It is the aim of this book to aid young students in gaining a comprehension of the essential principles of correct thinking.

It aims also to assist those teachers who find the use of a text-book in this subject advantageous. The book contains little that is original. It follows the main tradition of logical doctrine, with such omissions and changes in method as I, during years of experience, have found it desirable to make in an elementary course in logic.

The more important deviations from traditional logic will be found in the treatment of Judgment and the Syllogism and in the part entitled The Logic of Science. I have emphasized the distinction between Judgment and Propositions by giving to the former a separate exposition, preceding the discussion of Propositions.

In the treatment of the Syllogism, some technical matter, particularly that pertaining to moods, has been omitted, and changes have been made in the arrangement of topics which it is hoped will facilitate the comprehension of this subject. I hope I have made some improvement in the treatment of what is customarily called Inductive Logic, or Induction.

I have tried to make clearer the nature of Science and the limits of its explanations; and I have tried to give a more definite statement of the special problems of Science and to explain more clearly the methods of scientific thinking than is done in many books on this subject.

I am glad to acknowledge special indebtedness, both in the growth of my views and in the preparation of this book, to F. H. Bradley, Jevons, Bosanquet, Sigwart, and, preëminently, to John Stuart Mill, whose great work is, I think, still the most valuable contribution to Inductive Logic that has yet been made.

WILLIAMSTOWN,
March 19, 1906.

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CHAPTER I

INTRODUCTION

SECTION 1

THE MEANING OF LOGIC

LOGIC is the science of thought. Its aim is to ascertain and apply the principles which are regulative for right thinking. By thinking in logic is meant the familiar mental operations of forming ideas, or notions, making assertions, and reasoning. By right or valid thinking is meant that mode of thinking which attains its end, which is consistency, truth, and knowledge. The subject-matter of logic being thought, this science is not primarily concerned with things or facts of experience; it is concerned with the knowledge of facts only so far as that knowledge is attained by thinking.

The aim of all serious thought is knowledge; and logic aids the attainment of knowledge in two ways:

(1) It affords a negative criterion of truth or test of knowledge; for whatever violates the laws of thought cannot be true in any real world.

(2) Logic aids knowledge by defining the principles and laws of thought in accordance with which all knowledge beyond mere sense perception is attained.

If I am to reach knowledge, my thinking must conform to certain principles and laws; and it is the function of logic, as we have said, to ascertain and apply these constructive principles.

SECTION 2

DIVISIONS IN LOGIC

The division of logical doctrines should be based upon a difference between two aims of thought, rather than upon the two forms of inference known as deductive and inductive.

These two kinds of inference are associated with these two distinct fields of logic, but the difference between them is not the basis upon which a division of logical doctrine should be made. Logical thinking aims at two things: consistency and knowledge of fact. By consistency we mean that connection between a given judgment and other judgments which makes a judgment true if the given one is true, and false if the given one is not true.

Consistency means, if I think and assert this, I must also assert that. By truth in matters of fact, I mean agreement or correspondence between thought and fact; and the certainty of this agreement is knowledge. This difference in the aim of our thinking is the true principle of a division of logic. In accordance with it we have the Logic of Consistency and the Logic of Science. The former is customarily called Formal or Deductive Logic; the latter, Inductive Logic.

The function of Formal or Deductive Logic being to establish consistency in our thinking, the function of Inductive Logic the attainment of knowledge, I accordingly divide this study into two parts: —

Part One will deal with the Logic of Consistency; Part Two, with the Logic of Science.

PART ONE

THE LOGIC OF CONSISTENCY

CHAPTER II

THE CONCEPT

SECTION 3

THE NATURE OF THE CONCEPT

IN formal logic it is customary to present the doctrine of thought under three principal topics: The Concept or Names or Terms, Judgments or Propositions, and Inference.

I shall adhere to this division and order of topic, and proceed first to the Concept.

DEFINITION

A *Concept* is a mode of thinking in which something is by our thought treated as one thing and distinguished from all other things. In the objective sense of the term, a concept is anything which can be thus thought as one thing, identical with itself and different from other things; unity and distinctness from other things are consequently the defining characters of the concept in the objective meaning of the term.

The subject-matter, or content, of a concept is its meaning. The features by which this meaning is distinguished from other meanings constitute the marks, properties, or attributes of a concept. The sum total of these defining marks which are essential to the meaning of a concept are what is meant by its intension, or connotation.

I have chosen the word *Concept* in preference to Name or Term, which are more commonly used, because concept makes more prominent the mental act or logical process that is being studied; term or name designates properly that which is the product or result of this act. It is regrettable that there is no good word for naming this mode of thinking. Concept in present usage is ambiguous; it is not always easy to determine whether it is the mental operation itself, or the thing thought about that is meant. The words *subjective* and *objective* are the best we have to mark this difference in the meaning of the term; concept in the subjective sense being the mental act, and concept in the objective sense meaning the product of this act or that which is meant. It is in this objective sense I shall use the term unless I give notice of my intention to use it in the other signification.

Section 4

The Kinds and Distinctions of Concepts

1. **Simple and Complex Concepts.** — The content of a concept can be extremely simple, and its range very

limited; it can be something that is incapable of analysis or description, as, for example, this particular shade of blue I observe in the sky. This is a concept if, in addition to its being my momentary perception, I think it; if I judge respecting it that it is like the blue I saw in the waters of Lake Geneva; for by so doing I recognize this bit of color as one thing, distinguishable from other things. The content of a concept may be extremely complex, and its range of meaning illimitable. The universe itself may be a concept. Accordingly, considered in respect to their structure, concepts are *Simple* or *Complex.*

Simple concepts are those which do not admit of analysis or separation into other concepts. These concepts are formed at a stage of mental development which precedes the more conscious and purposeful thinking with which logic deals.

Complex concepts are those which can be resolved into other concepts. These concepts are the products of deliberate thinking. Such are the concepts of Science, Philosophy, and those for the most part which are employed in our ordinary intercourse with each other.

2. Concepts are again distinguished as **Universal, Individual,** and **Collective.**

A universal concept is formed by uniting in thought those features or marks and those only which are common to a number of individuals.

An individual concept is one which is formed by uniting in thought those marks which distinguish this

individual from all other individuals. In traditional logic a general name, answering to a universal concept, is a name which, in the same signification, is applied to all the individuals which constitute a given class or totality. A singular name is one which in the same signification can be applied to but one individual.

A collective concept is one which is formed by uniting in thought those marks or properties in virtue of which a number of individuals are considered, not separately or distributively, but as forming a single body or organism. For instance, in forming the concept army, I consider a number of individual men only so far as they are taken together and united in a particular form of organization; they thus form a single body. The individual men who compose an army are not considered distributively, but collectively; this collection implying both plurality and unity. Hence, this concept partakes of the character of both the universal and the singular concept. The collective name of traditional logic, like the general name, implies a number of individuals; but, unlike the general name, the collective name cannot be applied to these individuals taken distributively, but only as they are taken together. The name army, for instance, is not applicable to the individual soldiers which constitute it; the name horse is applicable to individual animals; but this name army is applicable only to that single organization which these individual soldiers all taken together constitute. The name army applies only to this unity of all the individuals so united.

3. **Abstract and Concrete Concepts.** — According to the point of view from which their subject-matter is regarded, concepts are *Abstract* or *Concrete.* The true distinction between these is best apprehended by observing two things in the formation of concepts.

The first is, that in forming any concept, particularly in forming the general concept, *some* abstraction is always involved, since this concept is formed by uniting only the marks which are common to a number of individuals, and there is consequently an abstracting from the other marks which belong as truly to these individuals as do the marks that are included in the concept. The difference between the abstract and the concrete concept does not lie in the different ways in which these concepts are formed; not in the fact that abstraction takes place in the formation of one concept and not in the formation of the other; for some amount of abstraction is involved in the formation of any concept.

The second thing to observe is that there are two ways of treating the subject-matter of any concept; we may either consider this subject-matter as something which possesses properties or marks, in which case we can consider these properties or marks only as they are conceived as belonging to this subject-matter; or, we may single out one or more of these properties, and by abstracting them, so to speak, from that to which they belong, form another concept. For example, in the concept of a Centaur, the various marks which constitute this concept, say a, b, c, d, . . . x, are

considered only as they belong to the something called Centaur; and this something is thought of only as possessing these properties. Now let us select one of these marks, say *b*, and, by abstracting from the other marks, we form a concept, — the concept of the particular property designated by this letter *b*; this concept is abstract, not because there has been abstraction in forming it, but because its subject-matter is considered apart from the Centaur of which it is a property.

Accordingly a concrete concept is to be defined as one in which the subject-matter is considered as something which has properties, and its properties are considered only as they belong to this subject-matter.

An abstract concept is to be defined as one in which the subject-matter,while it can be the property of something, is considered in abstraction from that something to which this property would belong; thus, whiteness is the property of some conceivable thing, but this property is considered apart from any subject-matter to which it can be attached; and when it is so considered, the concept is abstract.

This view of the distinction between concrete and abstract concepts explains the fact that many of the abstract names in logic are ambiguous, it being impossible to determine in the lists of such names given in text-books which are abstract and which are concrete. These distinctions as we have explained them are not fixed; some words doubtless are always names of concrete concepts, as horse, animal, man, state; others are perhaps as uniformly names of abstract concepts; but

very many such words are in some situations names of abstract concepts and in other connections they are names of concrete concepts. For instance, in most situations charity would be rightly called an abstract name or concept; but in the passage, "Charity suffereth long and is kind," the concept is concrete; it is so because it is considered and treated as a something which has properties; here the attributes are long-suffering and kind.

The same name can therefore be concrete in one usage and abstract in another; only the known intention and use of the author can determine in such cases whether the concept is abstract or concrete. Thus, in the proposition, "The mercy of the Lord is from everlasting to everlasting," mercy is an abstract concept; but in the sentence, "Mercy and peace shall go before him," the same name is concrete.

4. **Positive and Negative Concepts.** — According as the subject-matter implies affirmation or negation, presence or absence of this subject-matter, concepts are *Positive* or *Negative*. What it is to affirm or deny will be explained in the chapter on Judgments. The signs of negation are easy enough to recognize, and there should be no difficulty in distinguishing between positive and negative names. Such prefixes as not, non, invariably denote negative concepts.

Logicians are wont to distinguish a class of concepts which they call *Privative*. These, it is said, denote the mere absence of some state or quality which naturally or normally belongs to a subject; for example, *blindness*,

deafness. These terms, it is said, have no meaning if applied to beings which do not normally possess the powers of sight and hearing. On the other hand, the terms *not-seeing*, *not-hearing*, are negative; they are applicable to inanimate beings as well as to beings which can possess such powers. The difference between negative and privative is, however, one of degree only; a privative name is a negative name which has the additional implication that the subject ought normally to possess the specified mark or property.

5. **Absolute and Relative Concepts.** — In formal logic a name is called *Absolute* when its meaning is complete without involving a relation to another name. A name is *Relative* if its complete connotation does involve a relation to some other name. For example, the names husband, parent, brother, king, are relative names; while the names, metal, dog, man, happiness, are absolute names; the meaning of husband, parent, brother, king, is not complete without the relation to wife, other brothers or sisters, subjects; while the meaning of metal, dog, man, happiness, involves no such necessary relation.

The meaning of a relative name must not be confounded with the meaning which a name may have for any mind in consequence of all it suggests to that mind; in the logical sense of the term a name is not relative because it suggests various other names, every name does that; but because that which is suggested is a part of its meaning. The name home suggests to my mind a thousand things, no one of which is any

part of the logical connotation of this name; likewise the name father calls to my mind a hundred things which are not part of the logical meaning of this name; but this name does suggest one thing which is a necessary part of its connotation, viz. offspring, son or daughter; and apart from that relation and those two related things, this name has no logical meaning.

6. **Connotative and Denotative Names, Extension and Intension of Concepts.** — By some logicians names are distinguished as *Connotative* and *Denotative.* A connotative name is one which both designates a thing and implies attributes of this thing; man, for example, is a connotative name, since it means individual beings and implies certain attributes belonging to these individuals. A denotative name is one which points out or distinguishes some individual, and does not imply in its meaning attributes.

I have given the customary distinction; but it is not, I think, the right one. The real distinction is that of function or use of the name, or rather the intention of the thinker. In using the name man, the purpose of the thinker is to designate a certain group of qualities or attributes which are possessed in common by an indefinite number of individuals; these individuals are of importance, and are meant, only so far as they possess these attributes; it is the attributes and not the individuals that the name signifies, which constitute its connotation. The name John Smith is used to point out this individual, and to distinguish him from other individuals.

This distinction of connotation and denotation is not, properly speaking, a distinction in names themselves; it is rather a distinction in the purpose or intention of the one who is dealing with that which the names mean. The distinction which these logicians make is identical with that between universal or general, and individual or singular names.

We have seen that every name has a connotation; there are no denotative names in the sense in which these logicians use the term *denotative*, viz. a name which connotes no attributes. Even such individual names as John Smith must have some connotation; for it is only by virtue of some properties that I can distinguish this individual named John Smith, my John Smith, from other individuals, or from the John Smith that somebody else may mean.

The distinctions of *Extension* and *Intension* apply to the subject-matter of every concept, and it is better to make extension synonymous with denotation and intension synonymous with connotation. Accordingly, by the intension or connotation of a concept or name is meant the marks which are common to all the individuals to which the name applies; and by the extension or denotation of a concept or name is meant the individuals which possess these marks.

Every name has accordingly both extension and intension; this extension may be very great in the cases of names of classes or it may be reduced to a single individual in the case of a singular name. So with intension: it ranges from very few marks to an innumer-

able number of marks. A law of relation between extension and intension is sometimes stated in this way; extension and intension vary in inverse ratio, or the greater the extension of a name the less is its intension. A glance at the way in which general concepts are formed makes the truth of this statement obvious.

CHAPTER III

DIVISION, DEFINITION, AND CLASSIFICATION

SECTION 5

THE MEANING OF THESE TERMS

BY *Division* in logic is meant the separation of a given class into the lesser classes which are contained within it. By *Definition* is meant the specification or statement of the marks which distinguish these lesser classes within some larger containing class. By *Classification* is meant the systematic arrangement and distribution of individual objects so as to form classes.

Division and classification are complementary processes; and these processes cannot in reality be separated from each other; every division involves a classification, and every classification is at the same time a process of division.

A very intimate relation also exists between division and definition. A definition is involved in every process of division; and division is possible only as there is at the same time definition. I cannot systematically and completely divide a class without at the same time defining each of the successively formed classes;

nor can I define any one of these classes without at the same time dividing the larger class in which it is contained. It should also be observed that division and definition are closely related to extension and intension of names, division being the systematic statement or unfolding of the extension of a name, definition the systematic statement of the intension of a name.

Before proceeding to the exposition of these processes I have defined, it is necessary to explain a group of terms which occur in formal logic; they are the so-called predicables, *Genus*, *Species*, *Differentia*, *Property*, and *Accident*.

A genus is the larger class in the process of logical division; species are the lesser classes into which the genus is divided. Difference or differentia are the marks which distinguish the species from each other within a given genus. Property or proprium is a mark which belongs to every individual of a class, but which is not a part of the connotation or meaning of the name; thus, equality of its three angles to two right angles is a property of every triangle, but this mark is not essential to the meaning of a triangle, not necessary to its logical definition. By accident is meant a mark which belongs to some individual or to a part of the individual members of a class, but never to all the members of a class; for example, a diameter of six inches may be a mark of a certain circle, or of a number of circles; but it is not a mark of all circles.

There is a twofold distinction between property and accident: —

(1) A property is supposed to depend upon the essential nature or essence of the species or genus; while the accident does not. Thus, the equality of the radii of a circle depends upon the essential nature of the circle; hence it is a property of all circles; but a given length of a radius, say six feet, does not depend upon the nature of all circles; it is only an accident of certain circles or of one circle.

(2) The property mark must belong to every individual of the class; the accident cannot belong to every individual, and may belong to only one individual. The student must not understand that genus and species in logic designate fixed classes; they are, on the contrary, only relative distinctions. The same class can be a species relative to a larger class of which it is a part, and a genus relative to a lesser class into which it can be subdivided; thus, horse is a species relative to the genus quadrupeds, and a genus to the various kinds of horses into which we can divide this class.

Section 6

THE PROCESSES OF DIVISION, DEFINITION, AND CLASSIFICATION

From this explanation of the technical terms, we pass to the processes of division, definition, etc. We can best understand these processes if we study a concrete case. Accordingly, let us suppose we are to divide the books in a library. Our problem is both one of division and classification; for, at the outset, these books are a

class of things in virtue of certain marks common to them all. Now, it is obvious that different divisions and different classifications of these books are possible. The first step, therefore, in our task is the selection of a principle, or basis, of this proposed division and classification; in technical phrase a *fundamentum divisionis.* We will accordingly make the subject-matter of which these books treat the basis of the division, the *fundamentum divisionis.* We first divide the genus books into the following species; Histories, Books of Science, Literature, and Philosophy. This division is neither exact nor exhaustive, but it will answer our purpose. We note that the species are coördinate; the following diagram will show this: —

BOOKS

History *Science* *Literature* *Philosophy*

We note also that a peculiarity of the subject-matter treated in each of these species of books constitutes the differentia of each species: thus, the differentia of the history books is, that they treat of the actions of human beings in society.

Now let our division be carried one step farther, and we obtain the following new classes: History gives Ancient, Mediæval, and Modern History; Physical Science gives Physics, Chemistry, and Biology; Literature gives Fiction, Essays, and Poetry; Philosophy gives Logic, Theory of Knowledge, and Metaphysics. There results from this second step in our division the following things: —

(1) The lesser classes, which were species in the first step in division, now become genera in relation to the classes into which they have been separated; for example, Histories are a species of Books; but Ancient Histories are a species of histories, that is, Histories are both species and genus, species relative to the class Books, and genus relative to the class Ancient Histories.

(2) We note that a change takes place in the basis of division, or the *fundamentum divisionis*, in this second stage of the division. For the histories the basis of division becomes the period of time to which the historical phenomena belong; for the books in Physical Science, it is the special groups of natural phenomena which science comprehends.

(3) This process of division explains some distinctions which the student will find in text-books on logic; these distinctions are, summum genus, infima, and proximate species. The summum genus is the largest class with which a process of division begins, the largest class therefore in a system of division, or in a scheme of classification; the infima species is the class in which a complete division or any division terminates; thus, were I to complete this division of books, the last class of books would be infima species. By a proximate genus is meant the class that in the scale of division is nearest to the classes below it; and it should be added that these lesser classes are the proximate species of this genus; thus, in the division made above, Literature is the proximate genus to Fiction, Poetry, etc., but

not the proximate genus to English Poetry, or German Poetry. Had we subdivided literature into English Poetry, German Poetry, etc., we should have gone beyond or passed over the proximate species of literature.

Let us now return to the starting point in the division of these books, and make use of a different method or principle of division, a different sort of *fundamentum divisionis.*

Let the basis of division now be the possession or non possession of a given mark; and our division will be the following: Books are divided into Histories and non historical books, Histories into Modern Histories and those that are not modern; non historical books are divided into Literature and those which are not books on literature, etc. This method of division is technically called division by dichotomy, or dichotomous division, since it makes two classes only at each step of the process. This method of division has the advantage of being exhaustive at each step; since the class that has the negative mark includes all the individuals that are not put into the class which has the positive mark.

Let us turn now to the other closely allied process, definition; and, using the same case, the division of the books in the library, the problem now is to determine the marks or characteristics which distinguish each species of books. These marks define or bound off each of the several species into which the genus books was divided. If we examine the individual books which constitute only one of these species, say Histories,

we shall see that two sets of marks belong to them; the marks which make them members of the genus Books, and the marks which make them members of the species Histories; in other words, each history book possesses generic and specific marks.

A statement of both these marks is what the logicians mean by a definition: thus, Histories are defined as books which treat of the actions of men in society; and, since these specific marks are at the same time differentia, a definition consists in a statement of genus, species, and differentia. It must be kept in mind that it is only the generic and specific marks that are to be included in a logical definition. The property and the accident marks must never be given in such definitions. Definitions by property and especially by accident are serviceable for practical purposes, and are sometimes the only definitions possible; children for the most part define by such marks only; but such definitions are not admissible according to the canons of logic.

Another thing should be noted: the logical definition applies to a class of individuals, and not to any one individual as such; hence, only general names can be logically defined. The only way in which the individual can be defined is by a statement of all the marks which this individual alone possesses, and which consequently distinguish it from all other individuals; thus, if I were to define King Edward the Seventh, I should need to state every mark which this man alone possesses by which he is different from every other being in the universe. If I define King Edward by saying he is an

Englishman, etc., I state only those marks which he has in common with other men of this nationality; I define the species to which he belongs, not King Edward himself. Of course, we always do define to some extent individual or singular names, we must do so in order to distinguish individuals; but this process is not what the logicians mean by definition.

Section 7

Rules for Division and Definition

A number of such rules is usually given by logicians. The exposition of these processes I have given renders most of the rules laid down in text-books needless. It is an obvious corollary from the principle of division, that but one *fundamentum divisionis* can be employed in the same stage of the division; otherwise confusion of the species or cross divisions will result. For instance, had we divided books into Histories, books on Physical Science, and, say, octavos, we should have used two bases of division at the same time; and the consequence would have been a confusion of the subclasses; the species would have overlapped; there would have been cross divisions. Hence, a first regulative principle in division is, the *fundamentum divisionis* must not be changed during a given stage of the division.

Again, let me suppose that, in the division of those books, we had in place of books on Physical Science put Chemistries; by so doing we should have put a subordinate species in the place of a coördinate one; we

should have omitted a proximate species, and the consequence would have been that some of the books in our library would not have been divided or classified. Hence, a second regulative principle in division is, all the species in a given genus must be coördinate, or the genus must always be divided into its proximate species. It is obvious that a logical division should be exhaustive so far as it goes; that is, all the species which belong to a genus should be given; the division need not be complete in the sense of being continued until classes are reached which cannot be subdivided, but the division should exhaust each genus that is formed in the process of division. The student must not confound exhaustive division with complete division. It is no logical requirement that a division be completed; but it is a requirement that it be exhaustive.

For definition it is hardly necessary to give a special rule. A correct definition is, as we have seen, one which states all the generic and the specific marks, and only those marks. A definition which includes an accident mark is too narrow, and one which contains property marks also contains what is superfluous.

Section 8

Some Observations upon Division and Classification in Formal Logic

1. It is not unimportant to distinguish between the classifications of formal logic and scientific classifications. The aim of a scientific classification is to state

the actual relations of facts and phenomena in nature; and a classification is true, only if it conforms to the nature of those beings and phenomena; the classifications of logic are abstract, and, in relation to matters of fact, hypothetical. It is not necessary that a logical classification should agree with the objects and relations in *rerum natura;* it is only necessary that such a classification shall inwardly be coherent and consistent: a classification is conceivable which should be logical throughout, but which should agree with a scientific classification in no other circumstance save in being self-consistent and coherent. A scientific classification must, in order to be true, conform to logical principles; but a classification can be logical without at the same time being true. Classifications, therefore, which modern science rejects because they are untrue, are not less logical in their structure than are the classifications which science has put in their places.

2. The function of logic being regulative only, it does not teach us what are the properties and relations of things on the basis of which the classifications of science are made; it only teaches us how to construct a classification when we have found our things and classifying principles. Hence, the difficulties we encounter in our attempts at scientific classification are only in a small degree logical ones. They arise for the most part from our imperfect knowledge of the things themselves; and formal logic affords no remedy for this ignorance; since logic is not an organum of knowledge, but a doctrine of thought.

CHAPTER IV

JUDGMENTS

SECTION 9

THE NATURE OF A LOGICAL JUDGMENT

JUDGMENT is the mental act of perceiving and asserting a relation between two distinguishable things. There are two distinct operations in an act of judgment:

(1) the operation by which two things are distinguished and related to each other;

(2) the mental assertion that this relation thus recognized is a fact in the real world.

As the judgments we make relate to all sorts of matters, — things which exist in the sensible world, things that exist only in imagination, things perceived by sense, and things thought only, — so the two things which in judging are distinguished and related can be of the most various descriptions, a shade of color in a rose, the rose itself in the garden, the little flower plucked from the crannied wall, the universe of which that flower is a part, some action of an actual human being, some deed of a character in fiction, mathematical entities, the symbols of pure logic, A, B, . . . X, etc. We judge about all

such matters; and whenever we do so, these two things constitute the operation itself:—

(1) a relation between two things is perceived, apprehended;

(2) we entertain this relation with the conviction of truth,—in other words, our minds assert this relation; for, to have the conviction of truth and to assert, are one and the same thing.

To make this clearer, let us take some cases of judgment: I see a flower, and I judge concerning it that the color of its petals is deep crimson. Let us analyze this act of judgment. First, there is before me a certain object, a portion of the real world present to my senses; secondly, my thought notes and distinguishes in this total object or presentation, two things, — that which I have previously learned to be the petals of this rose, and a feature of these petals, the color of these petals, deep crimson; thirdly, I perceive a relation between these two distinguished things, rose petals and deep crimson color, — this relation is that of identity, the feature color I perceive in the petals of this rose is identical with that which I know as deep crimson; fourthly, I assert that this relation so present to my thought is a fact or is true. This which I call assertion as a mental act is just that conviction of truth or of reality of which I am conscious as I attentively consider this rose.

Again, suppose I am watching two boats, A and B, on a river; and I judge boat A is farther up the stream than boat B. Here, to my judging thought two objects are present, occupying a region of space in the real

world of sense perception; my thought distinguishes a definite relation between these two objects; that relation is in this case one of position in space. My mind asserts this relation; that is, it has the conviction that this relation between these two boats exists here and now in the real world.

Once more, let the case be that of the poet who said, "Truth crushed to earth shall rise again." The matters here do not belong to the world of perception; the judgment is about the abstract things of truth and victory over seeming defeat, matters of character and spiritual experience; but the same two essential thought operations are here discoverable; two things are distinguished and related — a condition of truth, viz. crushed to earth, and another conceived condition of the same truth, viz. rising again, victorious over seeming defeat. The relation between these two things we will call one of time, — the rising of truth from the earth is to follow her being crushed to earth. This relation is asserted in the mind of the poet; he feels the conviction of truth when he says these words.

These examples of judgment are sufficient to make clear, I trust, the nature of the mental operations that constitute it. The exact meaning of this all-important function will become more definite if we distinguish judgment from certain other mental acts and states.

(1) From exclamation. In pure exclamation there is no assertion. In place of assertion, there is a state of feeling, emotion, such as wonder, joy, sorrow, etc. When I exclaim Oh! Alas! Ah! and do no more, I have

not judged; I have not entertained anything as real; I have made no assertion about the real world; I have simply uttered a state of feeling.

(2) From question. A question is a mental state in which something is merely suggested, merely presented to thought, but not asserted. The conviction of reality is that which distinguishes a judgment from a question. So long as I must put an interrogation mark after my thought, I do not judge.

(3) From command. The essence of a command is the expression of will that something now conceived as possible become a fact; the essence of assertion is that something is *now* fact.

The distinctive feature of judgment, its differentia, seems to be this conviction of truth or of reality, of which any one can be conscious when that one judges. The subject-matter can be the same, whether I judge or exclaim or question, or command; the sole difference lies in my mental attitude to this subject-matter. When I judge, I claim for this subject-matter reality, a place in the real world; in the other mental states, I do not make such a claim for that about which I may be thinking.

Section 10

Judgment and its Verbal Expression

1. Judgment is a mental act which can take place only in the mind of some individual thinker; words are the signs to other minds that this act has taken place. It

is customary in logic to regard the sentence which expresses a judgment and the judgment as identical things. It is to be admitted that they are most intimately related, as thought and speech necessarily are; but to identify the mental judgment with the sentence which expresses it leads to misapprehensions.

One such error is the assumption that a judgment consists of parts as does the sentence; and that, like the sentence, it is formed by the combination of these parts; and consequently it is assumed that grammatical analysis and analysis of the judgment are the same thing. Now this conception of the judgment and its relation to a grammatical sentence is erroneous. To make in the case of a judgment the same division into parts which is made in the sentence is seriously to misapprehend the nature of judgment. The judgment does not consist of parts or elements as does a grammatical sentence; the judgment does not have a subject in the sense in which the sentence has a subject.

In the sentence, the subject is that about which something is asserted by the predicate; and this subject is a separable part of the sentence. In a judgment, if we speak of a subject at all, it means the entire subject-matter with which the judgment deals; for example, in the sentence, "The stars shine," the subject is stars; but the subject-matter of the judgment expressed in this sentence is that portion of the real world of sense perception which is named by all the words which compose this sentence. Thus, the subject or the subject-matter of a judgment includes both

subject and predicate of the grammatical sentence in which that judgment is expressed.

Again, in a sentence the predicate is that which is affirmed or denied of the subject. In a judgment assertion does not consist of affirming or denying one thing of another thing. In a judgment there is no such thing as a predicate in the grammatical meaning of that term. When I judge "Waters on a starry night are beautiful and fair," I do not assert one thing about another thing, viz. beautiful and fair about waters; what I mentally do is to assert that the relation of identity between what I mean by waters-on-a-starry-night and beautiful and fair is a fact in the real world. Grammatical predication and logical assertion, therefore, are different things.

2. What words and what sentences can express a judgment? The doctrine generally held is, that it is the declarative sentence only which can express a judgment. It is perhaps true that this sentence is the only way in which a judgment can be expressed without a possible doubt or ambiguity; but it is not true that in usage no other sentences do convey clearly, and with practical certainty, the fact that one has judged. Indeed, single words discharge this function; and, at the earlier stages of mental development, the single word is the only expression of judgment. When the child says "hot, burn, hurt," it has as unmistakably expressed a judgment, "This is hot, it burns, I am hurt," as if it had used a complete sentence.

The exclamatory sentence, though its primary func-

tion is to express feeling, does, nevertheless, make others certain that the exclaimer has judged as well as exclaimed. When one exclaims, "Oh, what a beautiful day!" there is as little doubt in the minds of those who hear, that a judgment has been uttered, as there is that an emotion has been expressed. The truth is, no exclamatory sentence expresses merely a feeling; only single words — Oh! Alas! etc., do that. The exclamatory sentence expresses both a judgment and a feeling state of some sort about the subject-matter of that judgment. If I exclaim, "Oh, what a pain in my foot!" "Oh, what a beautiful sunset!" I mentally assert the existence of pain, and the phenomenon of sunset; and I express at the same time certain feeling states which these objects excite in me.

The interrogative sentence, it must be admitted, may also express a judgment. There are interrogative sentences which ask pure questions; and a pure question is not a judgment; but there are interrogative sentences which do not properly ask questions in the logical signification of question. Such sentences express judgments as unambiguously as do declarative sentences; such are questions to which there is an expected answer, sentences in which information is not asked, but assent or denial is expected. The purpose of the questioner in such sentences is not to learn what the questioner does not already know, but to force some other person to accept a given judgment.

The imperative sentence is the only one which does not express a judgment. This sentence can only express a command.

SECTION 11

THE KINDS OF JUDGMENT

There are three kinds of judgment:—

(1) the *Categorical;*

(2) the *Hypothetical;* and

(3) the *Disjunctive.*

1. **The Categorical Judgment.** — In the categorical judgment the subject-matter is conceived and asserted as simple fact, as actual. For example, in the judgment "That cloud is dark," the simple existence of the cloud possessing a certain feature is asserted. In the judgment "All these men are honest," there is implied the actual existence of the individual beings called "these men."

2. **The Hypothetical Judgment.** — It is the distinctive feature of this judgment that it asserts, not an actual fact merely, but a connection between a supposed fact and something which follows from this supposed fact. The essence of the hypothetical judgment is, therefore, supposition, and the development of the consequence of that supposition; this judgment makes a supposition and develops its consequences. It is this feature which distinguishes the hypothetical from the categorical judgment; the difference is that between *actual* fact and *supposed* fact. The nature of the hypothetical judgment will be best understood from an examination of some concrete instances of its use.

As a first case let the judgment be, "If it rain, I shall remain at home." Here are two things: —

(1) a supposed fact, a supposed state of the physical world; and

(2) a consequence following from the supposed state of the world. Neither its raining nor my staying at home is regarded as actual; it is the connection between these two things that is asserted to be fact.

Again, take the judgment, "If he is asked a favor, he will grant it." I do not assert that he is asked a favor nor that the granting of that favor is to be a fact; but I do assert that these two things are so connected, that if asking the favor becomes a fact, the granting of that favor will become a consequent fact.

Every judgment, we have seen, asserts something about the real world; it is the very essence of a judgment to deal with reality, to claim truth. Now, in these two cases what is the reality that is asserted? Plainly, not the raining nor the being asked a favor, nor my remaining at home, nor his granting this favor; but a certain connection between the supposed things and something else; this is the real assertion made. Now, let us examine the assertion in each case, and we shall see that it means the following judgments:—

(1) The real world has such a constitution that if such an event as its raining in this particular region occurs, a certain other event, viz. my not going out, will exist.

(2) The real world is so constituted that given such a fact as this man being asked a favor, a certain other fact, viz. his granting this favor, will result.

The hypothetical judgment, therefore, just as the

categorical, deals with the real world; every such judgment asserts that the real world has a certain character or structure, so that if you assert or suppose a certain thing, some other thing will necessarily be a fact in this world. This interpretation brings to view another feature of the hypothetical judgment, and that is, this judgment depends upon an implied categorical judgment. Unlike the categorical, the hypothetical judgment cannot stand alone; it implies a categorical judgment; and the full meaning of this kind of judgment can be expressed only by adding an explicit categorical judgment. For example, the judgment, "If a body is allowed to fall freely, it tends toward the center of the earth," presupposes the categorical judgment, viz. the physical world is so constituted that in it there exists this necessary connection between a freely falling body and its direction towards the earth's center.

If, now, we compare the hypothetical with the categorical judgment, we note this further difference in meaning and use, — the hypothetical judgment is abstract; the categorical, concrete. I mean by this, that in judging hypothetically, we do not consider individuals or particular things as existing, but only as examples of universal laws or truths. The hypothetical judgment is used in asserting universal truths or laws of nature considered apart from any particular facts which illustrate or fulfill these laws; the categorical judgment, on the other hand, asserts what is true of these particular cases regarded as existing facts.

This difference between the two kinds of judgment in question is thus a difference between two ways of dealing with the real world, — dealing with this world abstractly and dealing with it concretely. I deal abstractly with my real world, when I treat the various particular things in it, not simply as facts in themselves, but as instances of universal laws. Thus, in the judgment, "If a body is heated it expands," I do not consider any particular heated bodies such as A, B, C, but a universal feature of the constitution of bodies. I deal concretely with the real world when I assert the existence in it of particular things or classes of things, provided I consider these classes as made of individual things which exist.

It follows from this difference in the nature and function of these two kinds of judgment that the hypothetical judgment always asserts what is universal. The categorical, on the other hand, while it can assert what is universal, asserts also what is particular, which the hypothetical never does.

This fact, that the categorical judgment asserts sometimes that which is universal, creates a difficulty in determining in some cases which of these judgments is made; for instance, is the judgment, "All men are mortal," categorical or hypothetical? In form it is categorical, but is it so in meaning? Does this judgment assert the mere fact that every individual man who has lived in the past, and every individual man now living, or who will ever live, will die; or does it assert a necessary connection between the nature or

attributes of man and the condition called mortality; so that, given an individual having the attributes of man, you are certain he has also the attribute mortality? I think there can be but little doubt that the latter is the meaning of this sentence. This judgment means, if an individual is a man, he will die. But take the judgment, "Every one present was delighted;" this judgment is both categorical and universal: it is categorical, because it distinctly implies the existence of the individuals themselves; it is universal, because it deals with a whole class of these individuals.

But now compare this universal with the universal in the preceding judgment, and we remark this difference in the universals: in the first case, the universal is made up of individuals who can be considered separately, and who can be counted or enumerated; in the second case, the individuals are not so considered. Only one feature or characteristic of these separate individuals is considered, viz. that by virtue of which they are each instances of a universal law. The existence of no one of these individuals is asserted or implied; it is the reality of a law only that is asserted.

We accordingly distinguish two universals, — the enumerative universal, and the universal of law or necessary connection of attributes. The enumerative universal is concrete; the universal of law is abstract.

This difference of universals enables us to determine in any case whether the universal judgment is categorical or hypothetical. If the universal is enumerative, the judgment is categorical; if, on the other hand,

it is the universal of law that is meant, the given judgment is hypothetical, albeit in form it may be the same as the categorical; thus, the judgment above, "All men are mortal," is a hypothetical judgment, because the universal here is that of law. The judgment asserts a necessary connection between the attributes connoted by the name man, and those connoted by the name mortal. Expressed in the form of a supposition, this judgment is, "If any being is a man, that being is mortal." On the other hand, the judgment, "All those present were delighted," is categorical, because it is an enumerative universal that is meant in this case.

3. **The Disjunctive Judgment.**— This judgment asserts that of two or more alternative possible things one is real. In this form of judgment, there are presented to our thought alternatives; and alternatives, if taken seriously, exclude each other, so that if one is taken to be real or true, the others are not real at the same time.

It must be noted that the disjunctive judgment does not assert which one of the suggested alternatives *is* real. It only asserts that *some one* of them is real. Thus, in the judgment, "A is either B, C, or D," the assertion is, that one of these alternative predicates goes with A; but it is not determined which one it is. It is implied, however, that if the real predicate is C, neither B nor D is the predicate of A at the same time. The disjunctive judgment, therefore, implies both knowledge and ignorance on the part of the one who judges.

This coexistence of knowledge and ignorance characterizes the situation in which one judges disjunctively. To judge, for instance, "A is either a knave or a fool, or a mixture of both," implies some knowledge of A's character; it implies knowledge enough to determine the possible predicates that can describe that character, — knavery, folly, or something of both; at the same time, one who thus judges about A's character confesses to some degree of ignorance, — the judge does not know which of these qualities constitutes A's character.

Two features, therefore, distinguish the disjunctive judgment from the other forms of judgment:—

(1) This judgment deals with alternatives, with possibilities which are so related to each other that if one of them is asserted to be real, the others are excluded from being real at the same time.

(2) There is implied in this judgment both knowledge and ignorance in the mind of the one who judges.

To the exposition I have given it may be objected, that there are disjunctive judgments which, instead of implying partial knowledge and partial ignorance, imply complete knowledge of the subject-matter, a knowledge so complete as to exhaust all the possibilities; such, it may be said, are the judgments of classification. For instance, when the geometrician says, "Triangles are either right-angled, obtuse-angled, or acute-angled," there is no ignorance whatever touching the subject-matter, but complete knowledge rather;

so that the classification is exhaustive. Two replies can be made to this objection: —

(1) Assuming that this is a case of genuine disjunctive judgment, one which presents real options or alternatives, there is something undetermined, some element of uncertainty, some degree of ignorance implied in this case; the geometrician does not know what sort of a triangle he will next meet with, and he does not know into which of the classes he will put the next one to which his attention may be drawn.

(2) But a better way of disposing of this difficulty is to point out the fact that this judgment is not really disjunctive in character; it is only a form of the categorical judgment, the peculiarity of which is, that in the form of a disjunction it merely divides the given subject-matter. There is no true disjunction in such cases; because no alternative possibilities are presented. This particular judgment, therefore, means that triangles are divisible into three classes; and that any given triangle belongs to one of these classes.

Section 12

The Quality of Judgments

In respect to their quality judgments are *Affirmative* or *Negative*. The distinction is that between affirming and denying, or negating. But, as we have seen, it is of the very essence of a judgment to affirm or assert; the conviction of truth, the claim of reality, is the nerve of every judgment. How then can there be such a thing

as negation, the denial of reality in a judgment? No conviction can be negative; it is present or it is not present; but if present at all, its nature is to claim reality, and this claim must be a positive thing. Nevertheless, negation is a fact in our thinking; and there are such things as negative judgments. Our problem then is to determine the nature of such judgments, the meaning of negation, and its function in our thinking.

The first thing we observe, if we examine a negative judgment, is, that this judgment is not independent; it cannot stand alone, it is possible only on the basis of some affirmative judgment; he who denies can do so only as he first implicitly affirms something; the spirit that merely denies may exist in the world of fiction, but not in the world of logical thought, or in the world of fact. He who says, "There are no living beings in the moon," if he claims that this judgment is true, must affirm that the real world as known by him excludes living beings from that body; he must be able to make positive assertions about the physical conditions of the moon in order to deny that there are living beings there. Every negative judgment therefore implies, and rests for support upon, some affirmative judgment. But more than this; we observe as the second fact, that negation has a positive use, and subserves a positive end in our thinking.

Our thinking is always seeking truth and knowledge; and the negative judgment is a stage in this progress of the mind toward knowledge. The special function of negation is to limit and define the direction in which our

thought is going, or must go, if it is to reach its goal. Let me illustrate this function of negation, this service of the spirit that denies. I will suppose a traveler is in search of a certain town, which he knows is upon one of a number of roads, all but one of which roads diverge from the road on which he is proceeding; I will further suppose that, at each point where a road diverges, a guideboard informs our traveler that the town he is seeking does not lie on that road. It is easy to see that these negative guideboards in the end give our traveler the information he desires; they keep him on the right road; they lead him to his goal; they define for the traveler the object of his search; they do so by successive eliminations of all the roads that lead elsewhere. In other words, the object of the traveler's quest is progressively defined to him; each of these eliminations of the wrong roads is a step toward the knowledge sought.

Now, this is just the function of the negative judgment. We set out in our thinking in search of a certain truth, a fact of some sort, just as our traveler seeks a certain place. This truth lies, at first, in different possible directions; by negation we exclude one after another these suggested possible directions; every excluded or negated one limits the number of remaining directions, until the last negation itself brings us to our goal, and gives us positive possession of the truth sought. Thus is negation a method of determination or definition, the peculiarity of this method being that it proceeds by elimination. I can define such a

subject-matter as B in one of two ways: I can either give the positive marks which define and distinguish B; or I can, by negative statements, eliminate one by one those which are not marks of B, and reach the same result, the definition or determination of B.

Section 13

Other Distinctions in Judgment

1. **Analytical and Synthetic Judgments.** — A judgment which analyzes a concept, or defines a name, is called by some logicians an analytical judgment, in distinction from judgments which unite to a subject something that is not already contained in the meaning of that subject. For example, the definition of a triangle, as a figure having three sides which inclose three angles, is, according to these logicians, an analytical judgment, because it only defines the meaning of a name, or analyzes the concept triangle; while the judgment, "The sum of the angles of a triangle is equal to two right angles," is synthetic, because in this judgment something not already contained in the meaning of triangle is asserted of triangles as its property. In other words, in this judgment two distinct things are united, or synthesized, instead of one thing being analyzed. Doubtless there is a distinction between the two thought operations in these two judgments; but this distinction really exists within *every* judgment; in every judgment there are both analysis and synthesis; these processes are not separable in our thinking; there is never analy-

sis without synthesis, nor synthesis where there is no analysis.

It is, however, true that in a given judgment, one or the other of these processes may be preponderant, and so give to this judgment its distinctive feature. The primary purpose of a thinker may be to analyze given subject-matter; and when such is the case, there is a propriety in calling the judgment analytical; or, the main purpose of the thinker may be to determine the relation between things, and to proceed from one thing to another; in which case the judgment is fittingly called synthetic.

2. **Modal Judgments: Judgments of Fact, Judgments of Necessity, and Judgments of Possibility.** — These distinctions are made by logicians; and they are generally assumed to correspond to differences in the mode of the assertion. *Assertorial*, *Apodictic*, and *Contingent* are also terms that designate these distinctions in judgments.

The assertorial judgment makes a simple assertion of actuality; for example, "Stars shine," is an assertorial judgment. The apodictic judgment asserts something to be necessary, or necessarily true; thus, "Things equal to the same thing must be equal to each other." The contingent judgment asserts what is possible; for instance, "There may be living beings on the planet Mars."

Let us examine these distinctions. And first, that between the assertorial and the apodictic judgment. Here are two judgments: "All the radii of a circle are

equal." "All the radii are necessarily or must be equal." Now, wherein lies the difference between these two judgments? They both have to do with the same subject-matter. The conviction of truth expressed in the second is not more complete or stronger than the conviction expressed in the first.

When the geometrician says, "All radii of a circle *must* be equal," he is not surer of that fact than he is when he merely says, "All radii of a circle *are* equal;" he does not mean to add anything to the strength of his conviction by saying, "All radii *must* be equal." Yet there is a difference in the two judgments, or rather in the mental situations which are reflected in them. This difference is not strictly in the judgments themselves, as we have seen; it lies in something which is implied and not expressed; and this implied thing is connected with the second judgment, the apodictic one; this judgment contains a reference to something beyond itself; this is the force of the word *must*. When I assert, "All radii must be equal," I imply some kind of reason or ground on which this assertion rests.

Further on we shall see that such judgments imply inference; here, it is sufficient to observe that the distinctive feature of the apodictic judgment is, that it implies a reason or ground on which its truth or validity rests; when we assert that something is necessarily a fact we imply that this fact is a consequence of, or is supported by, some other fact.

Let us examine next the so-called contingent judgment. We shall first observe that the words *may* and

can which are signs of this judgment, are ambiguous. They are used to denote a state of uncertainty in the mind of the thinker, and they are used to express the possibility of that which is the subject-matter of the judgment. Thus, when I say of a man who has been generally unsuccessful, "He may succeed, and he may fail again," I mean to express my uncertainty as to the issue of his next venture. In this situation, I do not judge respecting this man's business ventures; my judgment relates to my state of mind, and by implication I assert that I am not certain which of the two possible results, success or failure, is to be fact.

Now take the judgment, "The planet Mars may be or can be inhabited by beings like ourselves;" this judgment asserts a possible fact, not a doubting state of mind in the person who judges.

We should also observe another meaning of *may* in propositions. For instance, a teacher says to his class, "The class may omit chapter four in preparation of the next lesson;" this judgment asserts neither uncertainty nor possibility; it asserts rather permission; its meaning is, the omission of chapter four is a permitted fact. The teacher really asserts a state of his own mind, a state of willingness in reference to a certain possible action of those students.

Here, then, are three species of judgment, indicated by the words *may* and *can;* a judgment which asserts mental uncertainty, a judgment which asserts possibility, and a judgment which asserts consent or willingness. It is the second of these that is the contingent judgment of

the logicians, the judgment of possibility. Accordingly, let us next examine this judgment.

Every judgment, we have seen, deals with the real world; every judgment asserts fact. How, then, can there be a judgment of mere possibility, a contingent judgment? We saw in the case of the hypothetical judgment, that it presupposes, as its basis, a categorical judgment. One can suppose something, and draw therefrom a consequence only in a real world. It is just so in the case of a contingent judgment: this judgment presupposes a categorical one; it assumes a real world of a definite constitution. Possible things are thinkable, and can be asserted only in a world that is actual. When, therefore, the judgment is made, "Rational beings may inhabit the planet Mars," there is implied the judgment, "The physical universe has the sort of constitution which permits rational beings on the planet Mars."

This, then, is the meaning, the distinctive trait of the judgment of possibility; like the hypothetical and the apodictic judgments, it implies and is dependent upon another judgment. Contingency, possibility, are things which we can rationally think only as we think something as actual, or real.

CHAPTER V

THE LOGIC OF PROPOSITIONS

SECTION 14

THE MEANING OF THE PROPOSITION

IN formal logic, a proposition is a sentence which expresses a judgment. A proposition consists of two parts, technically called the two terms, or names; a third constituent, the copula, is recognized by most logicians.

A better analysis of the proposition is, to recognize in it two terms and a relation between them, these making the three elements of the proposition. The student must not identify the two terms of a logical proposition with the subject and predicate of the grammatical sentence, nor suppose that the grammatical copula is identical with the relation between these two logical terms. The various parts of speech which grammar teaches us to distinguish in a sentence have no existence in the logical proposition. Either term in the proposition may consist of a single word, or any combination of words; and any word, no matter what part of speech it may be, noun, adjective, preposition, etc., can form the subject or predicate term of a proposition.

These two terms in the proposition express the two concepts which are united in the judgments. And the assertion of the proposition is, that a definite relation exists between these terms. These relations are of all sorts, — space, time, quality, cause, likeness, difference, etc.; but, as we shall have occasion to explain later, for certain purposes in the use of propositions, these relations can be reduced to one or two, either the relation of subject and attribute, or of class to class. Thus the proposition, "Heat expands bodies," is equivalent to either of these two propositions: "Heat possesses the property of expanding bodies," or "Heat belongs to the class of things which is characterized by the property of expanding bodies," that is, heat is one of the things which expand bodies.

Again, logical analysis must not be identified with grammatical analysis. The logical analysis of a proposition consists simply in distinguishing the two terms (for convenience called subject and predicate terms), and the kind of relation which is asserted to exist between these terms. The logical analysis of a sentence which is not already in the form of a proposition consists in reducing the sentence to the form of one or more simple propositions. In order to analyze a sentence logically, sometimes various changes are made in its wording, clauses being reduced to simple combinations of words, phrases to single words, verbs changed to other words or omitted altogether, as we shall show in the practical exercises under this topic.

SECTION 15

KINDS OF PROPOSITIONS

Corresponding to the three essential kinds of judgments, logicians distinguish three kinds of propositions: the *Categorical*, the *Hypothetical*, and the *Disjunctive* proposition. Since a sentence which expresses a judgment is, from the point of view of logic, a proposition, whatever sentence expresses a categorical judgment, is a categorical proposition, whether this sentence be declarative, exclamatory, or interrogative. For the same reason both the conditional sentence of the grammarians and the declarative sentence can be hypothetical propositions. The disjunctive sentence, the sentence with either, or, and their equivalents, is always a disjunctive proposition.

Two other propositions should be distinguished: propositions which contain words implying exclusion, and propositions with exceptive or limitation words. These propositions are called *Exclusive* and *Exceptive* Propositions. Some of their peculiarities deserve to be noted. The exclusive proposition is interpreted in two ways: either as equivalent to a single hypothetical proposition, negative in force, or as equivalent to two categorical propositions, the one affirmative and the other negative.

Take the proposition, "Only members vote;" this proposition is interpreted so as to mean that if a person is not a member, that person cannot vote; so interpreted, the proposition says nothing about members; it does not assert that any member votes, nor that

no member or that some member votes; nor does it necessarily imply that at least some members vote; the entire assertion is about those who are not members; and this assertion is that they are excluded from the class of voters. According to the other interpretation, this proposition is equivalent to the two following: (1) "Those who are not members do not vote." (2) "Some members at least do vote;" and according to this view, the proposition asserts that non-members are non-voters; and *implies* that some members *are* voters.

The first of these interpretations hardly seems admissible. A situation is hardly conceivable in which it would be said that only members vote, if in fact no members were voters, or had not the right to vote. The proposition limits the right of voting to members; such is the force of the term *only*. It would be meaningless to announce that only holders of certain tickets could occupy seats in a grand stand, if it were not the intention of the managers to admit to those seats any who *did* hold these tickets. The exceptive proposition is equivalent to two propositions; thus, the proposition, "All but five were drowned," means five were not drowned; all the rest of that company were drowned.

Section 16

The Quality of Propositions

Negative Propositions.—We have seen what negation is in judgment. It now remains to consider in what

way negation is expressed in propositions, and consequently what propositions are negative in their quality. A *Negative* proposition is one which asserts either an absolute difference between two things, or an exception to a rule or general statement; for example, "Birds are not mammals" is a negative proposition, because it asserts an absolute difference between these two classes of living beings. "Right is not might" asserts a complete difference between these two things. "Some mistakes are not culpable" asserts an exception to the rule or the general statement, that mistakes are culpable things. Some one asserts, "Every man has his price;" the reply is, "*One* man, Mr. A. has not his price;" in other words, Mr. A. is an exception.

The student must be admonished that not all negative words in a proposition are signs of negation; also that some propositions which contain no negative words are either negative themselves or imply negative propositions. As an instance of an affirmative proposition in which there are negative terms, take the following, "What is not an animal is not a man;" in this proposition both the subject term and the predicate term are negative; and yet the proposition is not negative, since there is no real negation in the judgment.

A proposition is negative only when the negation affects the assertion. In the above proposition it affects only the terms; the proposition "Animals are not men," is a negative proposition; for here the negation is in the assertion. As instances of propositions whose terms are positive but whose force is negative,

take the following: "Few shall part where many meet," "Few are acquainted with themselves," " Perchance for a good man some would even dare to die." These propositions are negative in force, although they contain no negative terms. I think every one would admit that they contain negation rather than affirmation; "the few that part" are an exception to the rule. The force of the judgment is most do not part in the situation described by the poet. The clear implication of the proposition, "Few are acquainted with themselves," is, that most are not acquainted with themselves. That is what the writer means to say in an emphatic manner.

In the last example there can be no question about the quality of the proposition. Paul meant to assert that men as a rule are not willing to die, even for a good man. It should be noted that the hypothetical proposition is negative only when it is the connection between the protasis and the apodosis that is denied. For instance, the proposition, "If he did not commit this crime, he is safe," is affirmative; since the negation affects only the protasis; on the other hand, the proposition, "If he did commit this crime, he will not escape," is negative; because it is the connection between the protasis and the apodosis that is negated.

Finally, it must be borne in mind that the disjunctive proposition can never be negative, since in that case disjunction or alternatives would be impossible. To negate in the disjunctive proposition is to destroy the proposition itself. The proposition, "A is neither B

nor C," is a negative proposition, but it is not a disjunctive proposition, for the reason that it really presents no alternative.

Section 17

THE QUANTITY OF PROPOSITIONS, QUANTIFICATION

The *Quantity* of a proposition means the extent or scope of the assertion; and, since two terms constitute a proposition, quantity applies to both terms, and not to the subject term only, as formal logicians for the most part maintain. In its application to the subject term, quantity signifies the extent or amount of that which is considered in the assertion, or of that to which the predication applies.

In its application to the predicate term, quantity means the extent to which the predicate term is applied or used in the predication; thus, in the proposition, "Fixed stars are self-luminous," the predicate term *self-luminous* is not applied in its full extent to this subject; because, while the quality of being self-luminous belongs to all fixed stars, it may also be a quality of other bodies; but in the proposition, "Only the brave deserve the fair," the predicate term, *deserve the fair*, is applied in its whole extent, because this quality is limited to the brave.

In formal logic, propositions are distinguished as *Universal* and *Particular*. This distinction is based upon the quantity of the subject term only. A universal proposition is, accordingly, one in which the

predication applies to the entire subject term. A particular proposition is one in which the predication is applied to less than the entire subject term. According to this distinction the singular and the collective propositions are universal in their quantity, because the predication is applied to all that is named by their subject terms; for example, "The North Pole has not yet been reached," is a universal proposition.

The signs of universality in propositions are such words as *all*, *each*, *every*, *the whole*, etc. The signs of particularity are such words as *some*, *few*, *most*, *many*, *a part of*, etc. These signs must not be confounded with other words which imply generality or particularity, such as *always*, *ever*, *never*, *sometimes*, *rarely*, etc. These words do not affect the quantity of the proposition; for example, the proposition, "All men sometimes do wrong," is universal; while the proposition "Some men are always in the wrong," is a particular proposition; since the words *sometimes* and *always* affect the assertion, and not the quantity of the subject and predicate terms. The first proposition asserts that doing wrong sometimes is a quality or mark of all men; the second proposition asserts that being in the wrong at all times is the quality or mark of some men.

There is another error against which the student must be admonished: the confusion of the quantity of the subject term with that of a larger class or whole of which this subject term even taken in its full ex-

tension is a part, and which the subject term implies or suggests. For instance, "All male citizens have the right of voting," is a universal proposition; although its subject term clearly implies a larger class in which the class, all male citizens, is included, but in the proposition it is the male citizens only which are considered and to which the predicate, have the right to vote, can be given; none of the other inhabitants are considered in the proposition.

Quantification of Propositions. — So far I have set forth the customary doctrine which limits quantification to the subject term, and determines the quantity of the proposition by the quantity of this term only. But quantity is not a feature of the subject term only; it belongs to the predicate term as well; and, since in a proposition both terms must be considered, it is impossible to disregard this quantitative aspect or significance of the predicate term, when we are determining the exact scope or range of the assertion. For this reason it is no less important to consider the extension of the predicate term than to consider the extension of the subject term. In determining the exact scope of the proposition, "All men are mortal," it is not unimportant to consider whether this attribute of mortality is limited to men, or whether it may be also a quality of other beings.

By the quantity of a proposition, therefore, I shall mean that scope or extent of the assertion which is determined by the extension of its subject and predicate terms. Quantification is a method of treating a

given proposition so as to make explicit its quantity in the sense defined above. This method is artificial, and to subject propositions to it, is in some cases to create quite unnatural and rather awkward forms of statement; but this method is justified by the result attained, which is the exact determination of the meaning of the proposition.

Quantification is most easily effected if we conceive the relation between subject term and predicate term to be a relation between two classes; this relation in any given proposition will always be one of inclusion of the subject class within, or exclusion from the predicate class; and this inclusion or exclusion will be of the whole class, or of a part of it only. Making, then, the class relation the basis of quantification, we can easily see that the different relations as respects quantity which can exist between the subject class and the predicate class are the following: —

(1) All of the subject class may be included in the predicate class, so as to exhaust that class, leaving no room in it for any other class.

(2) All of the subject class may be included in the predicate class, yet so as to leave room for other classes, the subject class forming only a part of the predicate class.

(3) Some part of the subject class may constitute the whole of the class named by the predicate term.

(4) Some part of the subject class may be included in the predicate class, yet so as to leave room for other classes.

(5) Only some part of the subject class may be included in some part of the predicate class.

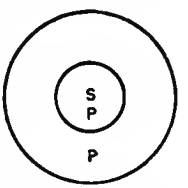
(6) All of the subject class may be excluded from the predicate class.

(7) Some part at least may be excluded from the predicate class.

(8) Only some part of the subject class may be excluded from the predicate class.

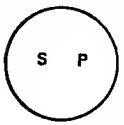
A convenient device for exhibiting these different possible relations between the subject and predicate classes is to use circles to represent the two classes, and the letters *S* and *P* to distinguish the subject and predicate classes. The different arrangements of these circles, together with the different positions of the letters *S* and *P*, will symbolize the quantification of any proposition; for example, the following diagram exhibits the quantification of the proposition, "All men are some part of the class called mortals."

The arrangement of the two circles shows that while all of the circle *S* is within the *P* circle, it does not exhaust that circle; there is room for other circles within it. *P* is placed in two positions to show that while the circle *S* is coextensive with some part of the *P* circle, it leaves some part of that same circle unoccupied. Take as another illustration of this use of circles, the proposition, "All of A's times of being present in a certain place are identical with all of B's times of being present in that place;" to exhibit the quantification



of this proposition the circles and letters are placed in this way: —

The two circles are concentric, and of equal extent. Let us now use the letters S and P for the subject and predicate classes in the eight statements made above, and we shall get the following formulæ for expressing quantification: —



1. All of S is all of P.
2. All of S is some part of P.
3. Some of S is all of P.
4. Some of S is some part of P.
5. Only some part of S is some part of P.
6. No part of S is any part of P.
7. Some part of S is not any part of P.
8. Only some part of S is not any part of P.

The student will see that the following arrangements of the circles and the letters S and P symbolize these formulæ: —

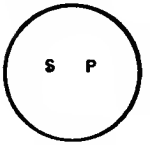


FIG. 1.

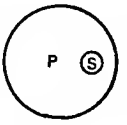


FIG. 2.

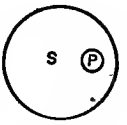


FIG. 3.

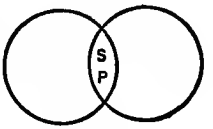


FIG. 4.

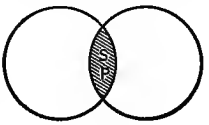


FIG. 5.

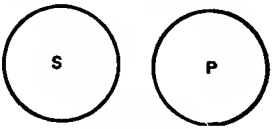


FIG. 6.

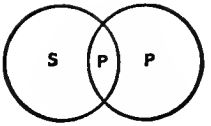


FIG. 7.

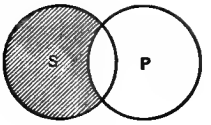


FIG. 8.

Inspection of these diagrams shows the following things which the student should keep in mind in constructing similar ones: —

(1) There are but four different arrangements of the circles themselves, as in figures 1, 2, 4, and 6.

(2) In formulæ 2 and 3 the positions of the letters *S* and *P* are reversed.

(3) In formulæ 5 and 8, portions of the circles are shaded, to indicate that it is only those parts of the circle to which the statements apply.

To make clear this matter of quantification, and to show the application of these formulæ, I will now quantify a few propositions.

(1) "All men love happiness." In this proposition the quantity of the subject term is already explicit, but not so the quantity of the predicate term; this does not show by its form whether all men make up the entire class of those beings who love happiness, or only a part of that class. Accordingly, I change the proposition in form so that it reads, "All men are at least a part of the class of beings that love happiness." In other words, I apply to this proposition formula 2, substituting the subject and predicate terms of this proposition for *S* and *P* of the formula.

(2) "*A* and *B* are always together." This proposition asserts that all of *A*'s times of being present are coincident with all of *B*'s times of being present. Substituting the terms of this proposition for the *S* and *P* in formula 1, we get "All of *A* is all of *B*."

(3) "Most men are honest." Applying to this proposition formula 4, it becomes, "Some men are some part of the class called honest beings."

(4) "Only some of those present took part in the sports." This proposition comes under formula 5, "Only some part of *S* is some part of *P*." The participation in the sports was limited to a part of those present; hence, formula 5 applies to this proposition.

(5) "Only members vote." If we make this proposition equivalent to two propositions, one of them will be, "Some members at least are voters," and the other will be, "No other persons are voters." The first of these propositions is quantified according to formula 4, and it becomes, "Some of the members are some part of the voters"; the second of these propositions comes under formula 6, and it becomes, "No non-members are any part of the class of voters." Instead of resolving this proposition into two others, we can quantify it under formula 3, and it will read, "Some, at least, of the members are all the class of voters."

(6) "It was only some of the candidates who did not pass in the examination." Formula 8 applies to this proposition; and the resulting proposition is, "Only some of the candidates are not any of those who passed in the examination."

These examples should sufficiently explain the method of quantifying propositions. The importance of this treatment of propositions can only be appreciated when we come to study the relation between propositions and the processes whereby we pass from one proposition to other propositions, as we do in reasoning. And it is to the study of these processes that we now proceed.

CHAPTER VI

INFERENCE, REASONING

SECTION 18

THE NATURE OF INFERENCE

INFERENCE is the act of proceeding from one or more given judgments to some other judgment. It is the acceptance of a judgment because of its connection with some other judgment already made. To infer is to believe something on account of its connection with some other thing. This something can be either a fact of experience, or an abstract idea or judgment.

1. **The essential elements of inference** are three:

(1) The datum or premise;

(2) The conclusion or result reached;

(3) The basis or ground of the inference.

The datum or premise is that from which the inference proceeds, its terminus *a quo*. The conclusion is the terminus *ad quem* of the process. The basis or ground is the reason or justification of the inferential process, or of the conclusion reached. The ground of inference must not be confounded with the datum or premise. Logicians have not always avoided this confusion; but

misconceptions of inference arise from the failure to distinguish these two things. The ground of the inference is not always, indeed is rarely, expressed; it is either implicit in a premise, as we shall see is the case in syllogistic reasoning, or this ground is a postulate or assumption that is tacitly made in proceeding from datum to conclusion, as in inductive inference.

Some examples will best make clear what I mean by this ground of inference and its distinction from the other two constituents of the inferential process.

Take as the first case the classic syllogism, "All men are mortal; Caius is a man; therefore, Caius is mortal." The first two propositions are the data or premises of this reasoning; the ground or justification of the inference that Caius is mortal lies in the two propositions which constitute the data. Examination of these two propositions discovers that this ground or reason for the conclusion that Caius is mortal, is the identity of the attributes of Caius and the attributes of all men, with which attributes mortality is connected. Caius is mortal because he has those human attributes which are connected with mortality.

As a second example, take the following: "*A* has died, *B* has died, *C* has died, etc.; hence, I infer that *X*, being a man, will also die." The ground of this inference is the postulate or assumption of uniformity of experience. This assumption is, that these cases of mortality of *A*, *B*, *C*, etc., are instances of some universal law; and, therefore, if *X* is like *A*, *B*, and *C*, he will die also.

2. **The Criteria or Tests of Inference.** — Inference must first be distinguished from other mental operations that on a superficial view seem to be the same. Inference is sometimes identified with association of ideas. This is a mistake; the mental operations are different. The characteristic of thinking is the discernment of relations between things. In thinking about things, we do not merely take note of them as events that occur; we do not merely perceive them as present facts, or recall them as past facts, or picture them as future facts; we perceive relations of various sorts between these things — relations of time, space, quantity, causation, etc.; and it is these relations themselves and not merely related things that are the subject-matters of our thinking. Mental association can deal with its objects only as concrete things, and as wholes; and it couples or connects its objects only in one way, that of conjunction, or succession in experience. Thinking deals with its objects differently; it analyzes them, abstracts qualities from things, relations from related things; and it unites things by relations quite other than those of mere associative connections.

Familiar instances will make clear this differentia of inference. A cat opens a door by jumping up and moving the latch. A dog, accustomed to go with his master in a boat, is told to get a sponge, goes to the house, and returns with it. A very young child puts its finger in a flame, gets burned, and next time avoids the flame. These are all situations in which a logical thinker would or could reason. The cat and the dog and

the child deal with data, with things which could be premises for a reasoner. They interpret in some fashion these data, and they reach conclusions or results of a practical character; and these conclusions could be formulated in propositions. It is quite certain that neither the cat nor the dog nor the child has performed an act of logical inference in these situations. All that is necessary to credit the cat with doing is, at most, a recall by associative memory of other instances in which it opened the door by hitting the latch; this hitting of the latch having been in the first instance purely accidental; the successful hits become associated with opening the door, and the cat acts upon this associative connection. The same explanation applies to the case of the dog. Accustomed to bringing things in response to words, gestures, etc., and accustomed to seeing water removed from the boat by the use of a sponge, the dog sees the water in the boat, does not see the sponge there, hears the sounds, sees the gestures which are associated in his experience with bringing objects, and brings the sponge. Here, as in the case of the cat, the mechanism of association, memory images, and motor reactions associated with them solve the problem. Mental operations of the same sort explain the action of the burned child that dreads the fire and avoids contact with it.

In these cases, while the mental processes lead to the same practical consequences that would follow from inference, they are not inferences; and for the following reasons: —

(1) There is in these cases no perception of the essential property or relation on which the consequent action depends. The cat does not perceive the essential property of the latch on which opening the door depends, nor does the dog perceive the property on which getting water out of the boat depends, nor does the child discern this property in the flame that burned its finger. Put a reasoner in these situations, and he would devise a way of opening the door if the latch were out of order; he would get a dipper, if the sponge could not be found; and he would avoid touching all hot objects as well as the flame.

(2) There is no genuine inference in these cases, because there is no consciousness of the necessary connection between data and result, between premises and conclusion. It is not seen that these consequences and no others will follow from the data or given premises. The cat does not mentally say, "If I do this, that must be the result." The dog does not virtually say, "If I get something that will hold water, that water in the boat can certainly be removed." But this is just what the logical thinker does say, viz. if a certain thing is done or is a fact, some other thing or fact will be the necessary consequence.

It is this discerned connection between premises and conclusion, and the acceptance of the conclusion solely because of this discerned relation it sustains to the premises, that constitutes an act of logical inference. For logic, there is no unconscious reasoning; the function of logic is to make us conscious of our

thinking; to think in the logical meaning of the term is to deal thus critically with our thinking, to be aware of what we are doing, and why we are doing it. One criterion, therefore, of genuine inference is that the conclusion is accepted because of its connection with the premises; and therefore we must be conscious of this connection.

This first differentia of inference will be better appreciated if we distinguish between inference as logic deals with it and inference in its psychological character or aspect. This difference is one of function and aim. From the point of view of psychology, inference, like other mental processes, is simply a fact or phenomenon to be described, and to be explained as science explains all phenomena. From the point of view of logic, inference is judged and valued according to its fitness to attain a certain end. The psychologist describes inference; the logician evaluates it. For the psychologist, it is a matter of indifference whether a given inference is correct or incorrect; for the logician, the character of the inference is a matter of fundamental importance. It is the function of psychology to describe the way in which inference takes place as mere mental process; it is the function of logic to ascertain how this process must go on, if it is to attain its end. Psychology is thus merely descriptive; logic is regulative or normative in its treatment of thinking. Psychology is not concerned with such matters as truth, validity, etc.; for logic, these aims and ideals are a primary concern.

Logical thinking marks an advanced stage in mental development; men reason, and reason correctly, long before they attain this stage of thinking. Most men who reason correctly do so without knowing why their reasoning is correct, just as people for the most part take food, and the right sort of food, without knowing how it nourishes them, or why one sort of food is good for them and another sort bad. The business of logic is to make one conscious in his thinking, to make one know when and why this thinking is correct. Here lies the practical function of logic. On its negative side, this function is to safeguard us from error; on its positive side, the function of logic is to guide us in the search for truth and knowledge.

The second criterion or test of genuine inference is that it must give as a conclusion something which is so far distinct from the data or premises, that it would not be perceived, but for the mental processes involved in inference. The conclusion must in some sense be contained in, or justified by, the data; otherwise it cannot be gotten from those data. The question is, *how* different from the premises must the conclusion be, to make the processes of obtaining it one of inference? Some logicians, John Stuart Mill, F. H. Bradley, and others, so emphasize this novelty in the conclusion as to reject the inference of formal logic for the reason, they maintain, that the conclusions in these inferences are not a distinct advance upon the premises, are not new facts which add something to the knowledge already had.

Hence, these logicians maintain that the syllogism of traditional logic is not a genuine form of inference, but only the appearance of inference. They say the syllogism begs the question, because it gives nothing in the conclusion that was not already contained in the given premises; the conclusion is only an explicit statement of what is implicitly asserted in the premises; it is like taking from a drawer something which you have first put there, or from a memorandum book some fact you have previously recorded there. The drawer is a useful thing, the memorandum book a serviceable thing; but they do not add to your possessions or to your store of known facts. Inference, say these logicians, must bring to light a new fact, must yield a judgment that is not already contained in the judgments from which it is derived.

Now these logicians are right in maintaining that there must be a real difference between conclusion and data. It is concerning the character and amount of this difference that the controversy between Mill and the defenders of traditional logic is waged. Mr. Mill, I think, exaggerates the amount of this difference; he misapprehends the nature and the principle of syllogistic reasoning, and he fails to discern the true character of the inference which he accepts as genuine.

Let us examine again the process of inference. In the first place, it is clear that this difference between the premises and the conclusion cannot be absolute, or there would be no connection between them, hence, no inference at all. Inference implies some continuity in

thinking, and continuity involves some element that is identical throughout the process, otherwise the successive steps cannot be linked. Any case of inference, if examined, will show something that is identical in premises and conclusion, or some element of identity in the connection between them; this is as true of the instances Mr. Mill gives — of genuine inference — as it is of those he rejects as spurious inferences.

To take one of Mill's examples, "Peter, James, John, etc., are mortal; therefore Caius is mortal, or all men are mortal." Now, Mill does not maintain that the mortality of Peter, James, John, etc., is the logical ground of our expectation that Caius will die, or of the conclusion, "All men are mortal." The logical justification of this belief is, even according to Mill, the assumed identity of essential attributes possessed by these men, those who have died, and those we believe will die. Mill *does* say that the only reason we have for believing that Caius will die, or that all men will die, are the particular facts of observation, the death of Peter, James, John, etc. But in this statement Mill has confounded datum with the ground of inference. It is not these particular facts as mere particulars that justify this inference to other facts, but the assumption that these particulars are instances of a universal law of necessary connection between the attributes of man and mortality. The reason, therefore, for the belief that Caius will die is, that he possesses those attributes which are thus universally connected with mortality.

If Mill had not confounded datum with ground of inference, he would not have made this erroneous assertion. Mill came near the truth in making the uniformity of nature the foundation of inductive inference; but he misapprehended the connection between this principle and inference; and, as I shall point out later, he did not rightly understand the principle itself.

Now, let us take an instance of the reasoning Mill rejects. "All men are mortal; Caius is a man, therefore Caius is mortal." Mill maintains that we have here no genuine inference, because the conclusion only asserts in explicit form what is already contained in the proposition, "All men are mortal." Mill is quite right, if this proposition is the sort of universal that he assumes it to be, — the enumerative universal. If this proposition, therefore, merely asserts that every individual man who can conceivably be counted either has died or will die, such a universal does contain Caius; and the proposition in which it is the subject term does, as Mill contends, assert the mortality of Caius; and the proposition which professes to be the conclusion only says the same thing a second time. But Mill failed to distinguish between two universals which are used in the syllogism, and therefore he failed to distinguish between a genuine inference in the form of a syllogism and a spurious inference.

Now, the universal proposition in this argument asserts a law of connection between the attributes of man and mortality; and the next proposition asserts

that Caius possesses these human attributes; and the third proposition asserts a genuine conclusion, just as genuine a conclusion as the one Mill draws from the premises, "Peter is mortal, John is mortal." This syllogism, therefore, is not open to Mill's objection, that it goes in a circle or begs the question. There is, we admit, an unproved major premise in the reasoning; so is there something assumed and not proved in the inference Mill accepts. I maintain, therefore, that this syllogism gives us a genuine inference, because the conclusion is something which is not recognized and cannot be justified until there has been some thinking done upon the datum. I cannot say Caius is mortal until I see that he is a man. I first link man with mortality, then Caius with man, then Caius with mortality.

Let us examine next some cases of inference according to formal logic against which Mill's criticism seems better justified. "All *A*'s are *B*'s; therefore, no *A* is not *B*, or what is not *B* is not *A*, or some *B*'s must be *A*'s." Surely, with more justice can one say these are not cases of inference. Each one of the propositions, after the first, only asserts the same thing in a different form of statement; there is identity of meaning under differences of expression. But is this identity of assertion apparent without some constructive effort, some thinking that discovers the substantial identity in these propositions? Grant that these propositions are all implied in the original one, and that the processes by which they are derived from it are only ways of developing the implications of this datum. Still, I think we

must admit that these processes add something to that datum, to what we saw in it or took it to be at the outset.

How many young students in logic would be able to recognize the identity of meaning in this group of propositions without considerable mental effort? Do not the subsequent propositions add something to the content and scope of the original proposition, so that they are not a mere repetition of the same proposition, with slight differences in wording? I think the answer must be in the affirmative, and that the formal logicians are in the right in regarding these operations as inferences. Of course, such inferences are of a different sort, and are based upon a different principle from the inferences which Mill regards as the only genuine ones. And this fact naturally leads to our next topic.

Section 19

THE FORMS OF INFERENCE

It is customary in logic to distinguish two kinds of inference, — *Deductive* and *Inductive.* These two kinds of inference differ first in form. In the deductive inference, at least one of the premises is a universal proposition, and the conclusion is a proposition of less extension than the premises; hence the name deductive, which implies a leading down from a universal or general truth to a special case. In inductive inference the premises are particular propositions, and the conclusion can be universal or particular.

Inductive inference proceeds either from particulars to a universal or to other particulars.

The second difference between these two kinds of inference is more important; it is a difference in function and aim. The function of deductive inference is to establish consistency between judgments; the function of inductive reasoning is to attain knowledge of facts. Consistency is the aim of deductive reasoning, as it is employed in formal logic. Truth and knowledge are the aim of inductive reasoning. For deductive reasoning, it is indifferent whether the premises are true or false propositions; its only concern is to reach conclusions which are consistent with the given premises, — true propositions if the premises are true, false conclusions if the premises are not true. Induction, on the other hand, starting with facts of experience, aims to enlarge our knowledge by reaching other facts not observed.

Connected with this difference in function and aim is a third difference between deductive and inductive inference, — difference in the subject-matters with which they deal. The subject-matters in which deductive reasoning is employed are abstract things for the most part. The subject-matters about which inductive reasoning is employed are concrete things; objects of perception are its data, facts of actual or possible experience are its conclusions. Hence, deductive inference belongs to formal logic; inductive inference, to science.

But there is a fourth and a more profound difference

between these two kinds of inference; it is a difference which has to do with the inferential processes themselves, with the nature of the connection between premises and conclusions, and with that which I have called the ground of inference. The conclusion in inductive inference goes beyond the premises to an extent and in a way which is not the case with the conclusion in deductive inference; thus, when I conclude from the fact that several substances exhibit a certain property, that all other substances which are like these will exhibit this same property, this conclusion goes directly beyond these data of experience. It is a step from the known to the unknown, as Mr. Mill rightly says. On the other hand, when, from the two propositions, "*A* is *B*," "*C* is *A*," I conclude "*C* is *B*," there is no such going beyond my premises; there is no step from the known to the unknown, as there is in the first case.

Again, the connection between premises and conclusion is of a different sort in these two inferences. The proposition, "All other substances which are like these known substances will behave in the same way," is not connected with the premises from which it is drawn, in the same way as is the conclusion, "*C* is *B*," with its premises. In other words, I believe the conclusion in the first inference for a different reason than the reason which compels me to accept the conclusion in the second inference.

The reason why I believe that all substances not yet observed will behave as do these substances

which have been observed, is an assumption of some causal connection, some uniform way of acting on the part of nature. The reason why I am certain that *C* is *B*, is that, analyzing and putting together the two given propositions, I discover that the properties which make *C* and *A* are identical with the properties which make every *A* a *B*; it is this identity of attributes that links *C* and *B*; and I can therefore say the proposition "*C* is *B*" is contained in or is implicated in the other two propositions, and analysis discovers this fact. Accordingly, we can say deductive inference is inference by implication; induction is inference based upon assumed causal connection.

Finally, it should be noted that the conclusions in deductive inference are certain and necessary. In inductive inference, on the other hand, the conclusion is only probable. The explanation of this fact is found in the differences I have already explained. Deductive reasoning, since it deals with our thoughts and their connections, and since it aims only to make our thoughts consistent, must reach conclusions that are certain. Inductive inference, on the other hand, having to do with matters of fact, with things which we can know only through experience, being a step into the unknown, cannot give conclusions that are certain. In the world of experience, in the world of actual things and events, probability, not certainty, is the best we can attain.

Each of the two kinds of inference we have discussed presents two varieties. Deductive inference is either *Immediate* or *Mediate*. Inductive inference sub-

divides into generalization of experience or inductive generalization, and inference by analogy. The distinction between immediate and mediate inference is this: in immediate inference the conclusion is drawn from a single proposition; in mediate inference two propositions are necessary for the datum. Immediate inference is direct; it consists of but one step. Mediate inference is indirect; there are two steps in the process. The two varieties of inductive inference will be considered in Part Two, since they belong to the Logic of Science.

CHAPTER VII

THE FORMS AND METHODS OF DEDUCTIVE INFERENCE

SECTION 20

EQUIPOLLENCE

IMMEDIATE inference is based upon two distinct relations that exist between any two propositions, — *Equipollence* and *Opposition*. By the equipollence of two propositions is meant that they express essentially the same judgment, but under such different forms of statement as to require some reflection to recognize this equivalence of the two propositions. Thus, these two propositions are equipollent, "All men love happiness," "Some beings who love happiness are all men." One and the same judgment is expressed in these two sentences, but this identity is not at first perceived; its recognition involves some exercise of thought; it is a something that is reached by a process which starts with a datum, and we have seen that such a process is an inference. Hence, for example, from the proposition, "All men love happiness," we infer the truth of this equipollent proposition, "Some beings that love happiness are all men."

Equipollence is maintained in four ways: —

(1) By obversion,

(2) By conversion,

(3) By contrapositions,

(4) By added determinants.

I will explain these methods of immediate inference in the order enumerated.

1. **Inference by Obversion.** — This means a change in a given proposition in consequence of which a second proposition is obtained, which contains a double negation of the original one; thus, "No just acts are not expedient" is a double negation of the proposition "All just acts are expedient," and it is therefore the obverse of that proposition. A proposition is obverted by putting into it a double negation.

This double negation is effected, either by using a negative word in each of the terms of the proposition, or by placing both negatives in the predicate part of the proposition; thus, the proposition, "All just acts are honorable," is obverted by placing a negative in both its terms, and the obverted proposition becomes, "No just acts are not honorable." The proposition, "Some just acts are expedient," is obverted by putting the double negation into the predicate part; and the obverted proposition is, "Some just acts are not not-expedient," — not inexpedient things. It should be noted that when a negative proposition is obverted it becomes affirmative in quality. Thus, "No men are always happy" obverted, becomes "All men are not-happy beings."

Let the student carefully note the difference in the negatives in these two propositions: In the proposition, "No men are always happy," it is the *assertion* that is negative, the proposition is equivalent to the following, "No men are any part of the class of happy beings;" in the proposition "All men are not happy beings" it is the predicate term that is negated, and this is done by prefixing the negative word to the predicate term, joining the two by a hyphen. This proposition is then equivalent to the following, "All men are in the class of not-always-happy beings." The student must keep in mind this difference between negating an assertion and negating the terms of a proposition. Obversion is based upon the principle that two negatives are in effect an affirmative. The particular negative proposition is obverted by joining the negative to the predicate term.

The peculiarity of obversion is that it changes the quality of a proposition, but so as to express the same judgment. The student should note that in obverting the particular negative proposition, the obverted proposition contains but one negative, and that is joined to the predicate term. Thus, obverting "Some men are not honest," we get "Some men are not-honest beings," or "Some men are dishonest." Note also that the quality of this proposition is affirmative; it means "Some men are in the class of not-honest, or dishonest, beings"; whereas the original proposition is negative.

The following directions may be of service to the student in obverting propositions: —

(1) Obvert a universal affirmative proposition by using a negative in both its terms.

(2) Obvert a universal negative by giving the subject term the form it has in the universal affirmative, and join the negative word to the predicate term.

(3) Obvert a particular affirmative by using two negatives in the predicate, joining one of them to the predicate term.

(4) Obvert a particular negative by joining the negative word to the predicate term.

2. **Inference by Conversion.** — By conversion is meant a change in the position of the subject and predicate terms of a proposition, which gives a second proposition, having the same quality as the original proposition, but in which the subject and predicate terms of the original proposition have exchanged places. Thus, to convert the proposition, "Some men are happy," I change the position of the subject and predicate terms, making them exchange places, as it were, and thus obtain the proposition, "Some happy (beings) are some men," which expresses the same judgment as the original proposition. The student must bear in mind that conversion, unlike obversion, does not change the quality of a proposition.

Logicians distinguish three varieties of conversion, or three methods of effecting it, — *conversion by limitation*, or *per accidens*, *simple conversion*, and *conversion by negation* or by *contraposition*. This last species of conversion must, I maintain, be rejected, because, as I shall show, it involves a change in the quality of the

proposition. Conversion by limitation applies to the universal affirmative proposition; and it consists in limiting the extension of the new subject term. Thus, to convert the proposition, "All men love life," I must not only exchange the positions of the subject and predicate terms, but the new subject term must be limited in its extension.

The converted proposition, therefore, is, "Some lovers of life are all men." Did I not make this change in the subject term, the two propositions would not be equipollent; for the original proposition means "All men are some of those beings that love life"; and to convert the original proposition without limiting the extension of the new subject term would give, "All lovers of life are all men," which may be a true proposition, but which does not certainly follow from the original proposition, nor does it express the same judgment as the original proposition.

Simple conversion is effected by exchanging the positions of the subject and predicate terms. This form of conversion applies to the particular affirmative proposition. Thus, the proposition, "Some *A* is *B*," converted, becomes, "Some *B* is *A*." The surest and easiest way of performing conversion is, first to quantify the given proposition, and then convert simply. For example, suppose we have the proposition, "A stitch in time saves nine"; first, quantifying this proposition we get, "A stitch in time is one of the things, or is some part of that which saves nine," then by converting simply, we obtain, "One of the things that save

nine is a stitch in time," or, "Some part of that which saves nine is a stitch in time."

I advise the student to follow in practice this one direction in converting all propositions, — quantify the given proposition and then exchange the positions of the subject and predicate terms.

3. **Inference by Contraposition.** — The contrapositive of a term is that term negated; the contrapositive of a proposition is a proposition which is obtained by obverting and then converting a given proposition. Thus, the contrapositive to the proposition, "All men love happiness," is, "Those who do not love happiness are not men." Hence, to obtain the contrapositive to any proposition, the simple direction is, first obvert the proposition, then convert this obverted proposition.

We have seen that the particular negative proposition cannot be converted, since to do so involves a change in its quality, which the principle of conversion does not permit. For example, were I to convert the proposition "Some men are not honest," according to the method of negation, I should obtain, "Some not-honest beings are some men," which is an affirmative proposition. The contrapositive to this proposition, however, can be given. Thus, obverting, it becomes, "Some men are some not-honest beings," and this converted gives, "Some not-honest beings are some men."

4. **Inference by Added Determinants.** — This mode of inference is effected by adding the same qualifying word to the terms of the given proposition. Thus, from the proposition, "All metals are elements," I can

infer that all heavy metals are heavy elements. But from the proposition, "Ants are animals," it would be wrong to infer that large ants are large animals. Hence, a caution must be observed in the use of added determinants. The added determinants must have the same meaning in both terms. This caution is not heeded when one infers from the proposition, "Poets are men," that bad poets are bad men. That which makes a man a bad poet does not necessarily make him bad as a man.

SECTION 21

INFERENCE BY OPPOSITION

We have seen what are the modes of inference which are based upon the relation of equipollence. Let us now examine the methods of inference which are based upon opposition, the other relation on which immediate inference is founded.

Two propositions may be opposed to each other in four ways: —

(1) As contraries,

(2) as contradictories,

(3) as subcontraries,

(4) as one subaltern to the other.

The strongest form of opposition is that between contraries; the least degree of opposition is that between a subaltern and the universal over it. The contrary relation is that between the universal affirmative and the universal negative, or that between two universals of opposite quality.

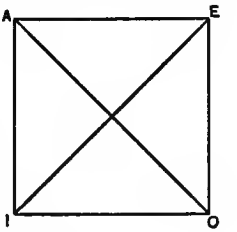
The contradictory relation is that between a universal of one quality and a particular proposition of the opposite quality; that is, between the universal affirmative and the particular negative, and between the universal negative and the particular affirmative. There are, therefore, two contradictories and one contrary.

The subcontrary relation obtains between the particular affirmative and the particular negative propositions, and the subaltern relation holds between each universal and the particular of the same quality. The following are examples of these various forms of opposition between propositions: The propositions, "All *A* is *B*," "No *A* is *B*," are contrary to each other. "All *A* is *B*," "Some *A* is not *B*," contradict each other; and so do "No *A* is *B*," and "Some *A* is *B*."

"Some *A* is *B*," and "Some *A* is not *B*," are subcontraries.

"Some *A* is *B*" is the subaltern of "All *A* is *B*," and "Some *A* is not *B*" is the subaltern of "No *A* is *B*."

In formal logic it is customary to represent these forms of opposition by the following diagram: —



In this diagram the letters are symbols of the propositions.

A the symbol for the universal affirmative.

E the symbol for the universal negative.

I the symbol for the particular affirmative.

O the symbol for the particular negative.

The upper side of the square represents the contrary relation; the diagonals of the square stand for the two contradictories; the lower side of the square, for the subcontrary relation; and the subalterns are represented by the two remaining sides of the square. It will be readily seen from this diagram that the contrary relation is that between *A* and *E* propositions; the contradictory is that between *A* and *O*, and between *E* and *I*. The subcontrary is that between *I* and *O*; and the subaltern that between *A* and *I*, and between *E* and *O*.

The student should accustom himself to the use of these symbols, and to the expression of the quantity and quality of propositions, and the various kinds of opposition between them, in term of these symbols.

1. Now, let us see what inferences are possible, taking any one of these propositions as a datum. We will begin with the *contrary relation*. The peculiarity of the contrary is this: if the given proposition is a true proposition, we can certainly infer that its contrary is false; but if the given proposition is not true, we cannot be certain that its contrary is true. For example, given the *true* proposition, "All men love happiness," I am certain that the proposition, "No men love happiness," is false; but if this first proposition is not true, I cannot be certain whether its contrary is true or false. The peculiarity of the relation between contraries is, therefore, that both cannot be true, but both can be false. Hence, the relation of contrary allows but one inference,

the inference that one of the contraries is false if the other proposition is true.

2. Examination of the *contradictory relation* will show that it permits two inferences; the peculiarity of this relation being, that if one of these propositions is true, the other is false, and conversely, if one is false, the other is true. Hence, two inferences are possible. Let me explain by an example. Let the given proposition be, "All men love happiness," and assume that this proposition is true; then it certainly follows that the proposition, "Some men do not love happiness," is false. Now, assume that the first proposition is false, then it will as certainly follow that the proposition, "Some men do not love happiness" is true. It cannot be false to assert that all men love happiness, unless it is true that some men do not love happiness.

3. The *subcontrary relation* makes it possible for both propositions to be true, but not possible for both to be false; thus, the propositions, "Some men are honest," and "Some men are not honest," can both be true; but they cannot both be false. The reason for this is obvious; take the first proposition, "Some men are honest," and, this being by supposition false, its contradictory, "No men are honest," must be true; and if so, the particular proposition, "Some men are not honest," must be true also. Hence, the subcontrary relation permits one inference only, the inference from the falsity of one of the subcontraries to the truth of the other.

4. Finally, the *subaltern relation* makes possible two inferences : the truth of the subalterns from the truth of the universals over them, and the truth of the universals from the falsity of their subalterns. For example, from the proposition, "All *A*'s are *B*'s," I am certain that some *A*'s are *B*'s; and likewise from the proposition, "No *A*'s are *B*'s," I am certain that some *A*'s are not *B*'s. But if these universals are false, I cannot be certain whether their subalterns are false or true; nor can I be certain that these universals are true, if their subalterns are true. Because some *A* is *B* it does not necessarily follow that all *A* is *B*. But let the subaltern be false, then will the universal over it be also false. If it is not true that some men are honest, it cannot be true that all men are honest.

CHAPTER VIII

MEDIATE INFERENCE, THE SYLLOGISM

THE syllogism, or mediate inference as the term implies, is inference by means of some intermediary conception or judgment.

SECTION 22

DESCRIPTION OF THE SYLLOGISM

The form of mediate inference is the *Syllogism*, which is a combination of two propositions from which a third proposition necessarily follows. The syllogism, accordingly, consists of three propositions, two of which are called the premises, and the third the conclusion. I shall first describe the syllogism in its regular forms. Of regular syllogisms there are three kinds:—

(1) the categorical,

(2) the hypothetical, and

(3) the disjunctive. I shall describe these in the order named.

1. **A categorical syllogism** is a syllogism the premises of which are categorical propositions. The syllogism contains three terms, called the major, the minor, and the middle terms. The major term is the term of greatest extension; and the premise in which it

occurs is therefore called the major premise. The minor term is the term of lesser extension; and the premise in which it occurs is called the minor premise. The middle term is so called from its function, which is that of uniting the two other terms; it is by means of this term that the major and minor terms are united in the conclusion; the middle term is therefore the connecting link between the other two terms.

The structure of the categorical syllogism can best be understood by an example. "All metals are elements; iron is a metal, therefore iron is an element." In this syllogism, the major term is *element*, since this is the term of largest extension; *iron* is the minor term; because it is, compared with the term *element*, a term of lesser extension; and the middle term is *metal*, because it is the term by means of which the minor and major terms are connected in the conclusion. The major premise in this syllogism is, "metals are elements," because this proposition contains the major term; and the minor premise is, "iron is metal," because this proposition contains the minor term. It should be noted also that the middle term, *metal*, occurs in each of the premises; and that both major and minor terms occur in the conclusion.

Figures of the Syllogism. — I have described the general structure of the categorical syllogism. I must next describe certain forms which this syllogism assumes according to the position and combination of its terms; these are the so-called Figures of the syllogism. The figure of the syllogism is the form which

is determined by the position of the middle term in the premises relative to the other two terms. Examination of a syllogism will make it evident that four arrangements of the middle term with the other two are possible: —

(1) the middle term can be the subject in the major premise, and the predicate in the minor premise;

(2) the middle term can be the predicate in both premises;

(3) the middle term can be the subject in both premises;

(4) the middle term can be the predicate in the first premise, and the subject in the second premise.

Using symbols for the three terms, P for major, S for minor, and M for middle term, we can represent these four arrangements of the terms and propositions as follows: —

FIG. I.	$M - P$ $S - M$ $S - P$	FIG. II.	$P - M$ $S - M$ $S - P$
FIG. III.	$M - P$ $M - S$ $S - P$	FIG. IV.	$S - M$ $M - P$ $S - P$

These diagrams represent the syllogistic figures usually designated as Fig. I, Fig. II, Fig. III, and Fig. IV.

It will be seen that the characteristics of these figures

are the following: In Fig. I, the middle term is the subject in the major premise, and the predicate in the minor premise. In Fig. II, the middle term is the predicate in both premises. In Fig. III, it is the subject in both premises. In Fig. IV, the position of the middle term is the same as its position in Fig. I, the only difference between these figures being, that in Fig. IV the order of the premises is reversed, the minor premise being placed first. Figure I was the typical syllogism of Aristotle, though he recognizes Figs. II and III, but as imperfect forms of the syllogism.

Moods in the Syllogism. — Mood is the form of the syllogism which is determined by the quantity and quality of the propositions that form its premises. Now, since there are four such propositions, distinguished by their quantity and quality, there can be as many moods in each of the syllogistic figures as there are different possible combinations of the four propositions, taken two at a time. For example, in Fig. I, there can be the following combinations: the universal affirmative or *A* proposition may be used for both premises; the universal negative or *E* proposition, the particular affirmative or *I* proposition, and the particular negative or *O* proposition can likewise be used. Each of these universals may be combined in two ways with each of the other propositions; and the same is true of each of the particular propositions. The following arrangements show the total number of such combinations, and consequently the total number of moods possible in Fig. I: —

1	2	3	4
A A A A	*E E E E*	*I I I I*	*O O O O*
A E I O	*A E I O*	*A E I O*	*A I E O*

Here are four groups, formed by combining each of the four propositions with each of the other propositions; and each group gives four combinations, the total number being sixteen; that is, there are sixteen possible moods in Fig. I. As each figure gives the same number of such moods, the total number of possible moods of the syllogism is sixty-four. But, as will be explained later, only about one fourth of these moods are valid syllogisms, that is, syllogisms which yield a conclusion.

2. **The Hypothetical Syllogism.** — A hypothetical syllogism is a syllogism the major or first premise of which is a hypothetical proposition, and the minor premise a categorical proposition. In this syllogism the major premise states a supposition or condition and its consequence or result. The minor premise asserts either the truth or the untruth of the supposition, or it asserts the same of the consequence stated in the major premise. The conclusion, according to the assertion made by the minor premise, affirms or denies either the supposition or the consequence. When this conclusion is an affirmative proposition, the syllogism is called *constructive;* when it is a negative proposition, the syllogism is called *destructive.*

The minor premise can make four assertions: —

(1) that the supposition made in the major premise is true,

(2) that this supposition is not true,

(3) that the consequence or result stated in the major premise is fact, or

(4) that this consequence is not fact.

We shall later see that in a valid hypothetical syllogism the minor can make but two assertions. These features of the hypothetical syllogism are exhibited in the following examples: —

1. If *A* is *B*, *C* is *D*;
   A is *B*;
   Therefore, *C* is *D*.

2. If *A* is *B*, *C* is *D*;
   C is not *D*;
   Therefore, *A* is not *B*.

3. If *A* is *B*; *C* is *D*;
   A is not *B*;
   No conclusion.

4. If *A* is *B*, *C* is *D*;
   C is *D*;
   No conclusion.

5. If *A* is not *B*, *C* is not *D*;
   A is not *B*;
   Therefore, *C* is not *D*.

6. If *A* is not *B*, *C* is *D*;
   A is not *B*;
   Therefore, *C* is *D*.

Two peculiarities of the hypothetical syllogism are shown in examples 5 and 6. In 5, although the minor premise is a negative proposition in form, it asserts the truth of the supposition made in the major premise, viz. that *A* is not *B*. In 6, an affirmative conclusion is drawn from what appears to be a negative premise; in reality, however, the major premise is an affirmative proposition, the negation in it not affecting the assertion.

3. **The Disjunctive Syllogism.** — A disjunctive syllogism is one having for its major premise a disjunctive proposition, and for its minor premise a categorical proposition, the conclusion of course being a categorical proposition. We saw that the function of the disjunctive judgment is to present alternatives. Accordingly, the major premise of this syllogism presents two or more alternatives; the minor premise accepts or rejects one or more of these alternatives; and the conclusion is affirmative or negative, according to the character of the minor premise. Logicians distinguish two moods of this syllogism, technically designated, *modus ponendo tollens* and *modus tollendo ponens*. These words are determined by the character of the minor premise; if the minor rejects or takes away an alternative, the conclusion is affirmative; we have then the *modus tollendo ponens;* if, on the contrary, it accepts an alternative, the conclusion is negative; and we have, therefore, the *modus ponendo tollens*. The following syllogisms illustrate these two moods: —

A is either *B* or *C*;
A is not *C*;
Therefore, it is *B*.

In this syllogism, since the minor rejects one alternative, the other alternative is affirmed; we have, therefore, a case of *modus tollendo ponens*.

A is either *B* or *C*;
A is *B*;
It is therefore not *C*.

Hence, we have the *modus ponendo tollens*.

Irregular Syllogisms. — Having described the syllogism in all its regular forms, I will now describe those deviations from the regular syllogisms which are very common in our reasonings.

These irregularities in syllogistic reasoning are produced in two ways: —

(1) by combining features of the syllogism in its different forms;

(2) by abridgment or condensation in a syllogistic argument, this abridgment being made either by omitting one of the propositions of a single syllogism, or by combining several syllogisms so as to form a chain of reasoning.

The irregular forms of reasoning thus produced are the following: —

(1) the Dilemma,

(2) The Enthymeme, and

(3) the Sorites. These shall be described in their order.

1. **The Dilemma.** — The dilemma is a form of argument in which two or more alternatives are so presented that a certain conclusion is inevitable, whichever of these alternatives is accepted or rejected by a disputant. These alternatives are the so-called horns of the dilemma; one or the other has to be taken with a damaging, if not fatal, result in either case. The form in which this sort of argument is presented is a syllogism having for its major premise a hypothetical proposition which presents at least two alternatives, and for its minor premise a disjunctive proposition; the conclusion of this syllogism is either a categorical or a disjunctive proposition.

This peculiar structure will be understood best by an example. Take the following argument: "Every law is either useless, or it occasions hurt to some person. Now, a law that is useless ought to be abolished, and so ought every law that occasions hurt; therefore, every law ought to be abolished." The argument is a dilemma. We have only slightly to change its form and we shall get a dilemmic syllogism. The major premise is, "If a law is useless it ought to be abolished, and if a law is harmful it ought to be abolished." The minor premise is, "Every law is either useless or harmful." The conclusion is, "Every law ought to be abolished."

Here is a typical dilemma; two alternatives are offered in the major premise, one or the other of these is accepted in the minor, and a certain conclusion follows. The two horns of this dilemma are

the uselessness of law and the harmfulness of law. Let either of these be accepted, and the same conclusion follows. Of course, the escape from this dilemma lies in challenging the major premise, in rejecting the alternatives, either on the ground that they contain unwarranted assumptions, or that they do not exhaust the possible alternatives. But this matter belongs to another chapter.

The dilemma is called *constructive* when it leads to an affirmative conclusion, and *destructive* when the conclusion established is negative.

Dilemmas are also *simple* or *complex*, according as the conclusion is a categorical or a disjunctive proposition.

I give the following examples which will make clear these forms of the dilemma:—

1. Simple constructive dilemma:—
   If A is B, C is D; and if E is F, C is D; but either A is B or E is F; therefore, C is D.

2. A complex constructive dilemma:—
   If A is B, C is D; and if E is F, G is H; but either A is B, or E is F; therefore, either C is D, or G is H.

3. A simple destructive dilemma:—
   If A is B, C is D; and if A is B, E is F; but either C is not D, or E is not F; therefore, A is not B.

4. A complex destructive dilemma: —
   If A is B, C is D; and if E is F, G is H; but either C is not D, or G is not H; therefore, A is not B, or E is not F.

Jevons, Whately, Mansel, and some other logicians reject the simple destructive dilemma, but without sufficient reason, I think. These logicians, I should add, teach that in the dilemma two distinct antecedents as well as two alternatives must be presented. Of course if their conception of the dilemma is the correct one, there cannot be a simple destructive dilemma, since such a dilemma does not have two distinct antecedents. I maintain, however, that the description of the dilemma given above is correct; and if so, the simple destructive dilemma is as admissible as is the complex destructive dilemma.

2. **The Enthymeme.** — An enthymeme is a syllogism with one of its propositions omitted. Any one of the three propositions may be the omitted one; but it is more common to omit one of the premises. The following will serve as examples of enthymemes: —

1. The English government is liable to frequent changes in its foreign policy, because it is democratic.
2. The English government is liable to frequent changes in its foreign policy, because all democratic governments are liable to such changes.
3. All democratic governments are liable to frequent changes in their foreign policies; and the English government is democratic.

Inspection of these examples discovers that in 1, it is the major premise that is omitted; supplying it, the complete syllogism is, "All democratic governments are liable to frequent changes in their foreign policies; the English government is democratic; therefore, the English government is liable to frequent changes in its foreign policy." In example 2, the student will perceive that a minor premise is to be supplied; and in the third example it is the conclusion which is not stated. The enthymeme, though an irregular construction, is the most common form in which deductive reasoning occurs, the complete syllogism being rarely met with outside of text-books on logic.

3. **The Sorites.** — A chain of syllogisms is a series so constructed, that the conclusions of one syllogism, either expressed or implied, form a premise for the next. Prosyllogism is the name given to a syllogism that proves the premises of the following syllogism; and episyllogism is the name given to a syllogism that follows and rests back upon a preceding one for its support. The syllogistic chain assumes two forms, determined by the way in which the syllogisms are combined to form the given chain. The first of these is the epichirema, of which the following is an example: "All M is P because M is X; All S is M because S is Y; therefore, All S is P." If we examine this peculiar combination of propositions, we shall find that they constitute a chain of syllogisms; for each premise in this single syllogism is an enthymeme, and the enthymemes expanded give the following chain: —

Whatever is X is P;
M is X;
Therefore, M is P.

Whatever is Y is M;
S is Y;
Therefore, S is M.

Whatever is M is P;
S is M;
Therefore, S is P.

The epichirema can, therefore, be described as a syllogism with supporting reasons for its premises, or as a syllogism whose premises are enthymemes. The more typical form which the syllogistic chain presents is the sorites, which may be defined as a chain of enthymemes.

There are two well-known forms of sorites, one called the Aristotelian, the other the Goclenian, from Goclenius, a German logician in the seventeenth century. The following is an example of the Aristotelian sorites: —

All A is B;
All B is C;
All C is D;
All D is E;
All E is F;
Therefore, All A is F.

The student will readily see that this chain is formed by combining syllogisms of Fig. IV, by omitting the

conclusion of each prosyllogism. If the omitted propositions are supplied, the chain becomes the following:—

A is *B*;
B is *C*;
Therefore, *A* is *C*.
A is *C*;
C is *D*;
Therefore, *A* is *D*.
A is *D*;
D is *E*;
Therefore, *A* is *E*.

It will be seen that in this chain the unexpressed conclusion of each syllogism is the minor premise of the succeeding syllogism; and that the subject of the first proposition is also the subject of the last proposition in the sorites.

The following is an example of the Goclenian sorites:

A is *B*;
C is *A*;
D is *C*;
E is *D*;
F is *E*;
Therefore, *F* is *B*.

The student should see in this chain a combination of syllogisms in Fig. I, in which the unexpressed conclusion of each prosyllogism is the implied minor premise of the syllogism following it. Another thing should be also seen: this sorites, if its propositions are

read in the reverse or backward order, is the same as the Aristotelian sorites. Thus, beginning with the last proposition but one, the chain becomes —

F is E;
E is D;
D is C;
C is A;
Therefore, F is A.

which is our own Aristotelian sorites. The real distinction between these two sorites is, that in the Aristotelian sorites the reasoning is progressive, while in the Goclenian sorites the reasoning is retrogressive.

SECTION 23

REGULATIVE PRINCIPLES AND RULES FOR THE SYLLOGISM

The syllogism has now been described in all its forms and varieties; it remains to ascertain the principles of valid reasoning, and the rules which must be observed in the employment of the syllogism.

A syllogism is valid when the conclusion follows from the given premises, irrespective of the truth or untruth of the premises in themselves considered. It is not, therefore, essential to the validity of a syllogism that its conclusion be a true proposition; but only that its conclusion follow from, and be consistent with, the premises; a valid syllogism being one which gives for its conclusion a true proposition if the premises

are true propositions, and a false proposition if its premises are not true. It should also be borne in mind that it is not the function of formal logic to find or to establish true premises; consistency, not truth, being, as has been said, the aim of this part of logic.

The Validity of Disjunctive and Hypothetical Syllogisms. — In examining the conditions and the rules of valid syllogisms, we can best begin with those which are of simplest construction, the disjunctive and the hypothetical syllogisms.

1. The *disjunctive syllogism* is based upon the principle of alternatives, which, as we have seen, it is the function of the disjunctive judgment to present. This principle requires that, if one or more of the given alternatives are accepted, the remaining alternative or alternatives must be rejected. From this principle we derive the following rule for the disjunctive syllogism: The minor premise must either affirm or deny some one of the given alternatives.

2. The *hypothetical syllogism* is based upon the principle of supposition and consequence, or antecedent condition and consequent. This principle is, that if the supposition is fact, or is true, the consequence is also fact, or is true; or, otherwise expressed, if a specified condition is fulfilled, its consequence is fact. We can therefore always infer the fact or reality of a specified consequence from the truth of the supposition or from the fulfillment of the condition. But this principle does not permit us to infer the truth of a supposition, or of a condition, from the truth or fact of

a given consequence; because, while the thing which is called a consequent must exist if its supposed condition exists, this thing might conceivably exist if that particular condition did not exist; hence, the mere fact of this consequence proves nothing.

This will be made clearer by an example. Take the following, "If it is cloudy, there will be no dew to-night." Now, suppose that we learn that it is cloudy, we shall then be certain that there will be no dew; but suppose we are told to-morrow morning that there is no dew, can we be certain that it was a cloudy night? Not unless we know that a cloudy night is the only condition of there being no dew; and the proposition does not tell us that. But now, suppose that we are told that there was dew; then we can be certain that the night was not cloudy. Hence, the principle of supposition permits us to infer from the nonexistence of the consequence, the nonexistence of the condition, or supposed fact. Accordingly, we derive the following rule for the hypothetical syllogism in its usual form, The minor premise must either affirm the condition, or deny the consequent.

There is, however, a permissible form of this syllogism which gives a conclusion when the minor premise affirms the consequent. The following is a case of this sort, "Only if *A* is honest can he gain my confidence; he has gained my confidence; can you not infer that *A* is honest?" The peculiarity of this syllogism is that the major premise states an exclusive hypothesis; the honesty of *A* is the sole thing,

which, if it exist, can give me confidence in him; and consequently if it is fact that *A* has gained my confidence, it must be fact that he is honest.

Our rule for the hypothetical syllogism must accept this qualification, viz., When the major premise contains an exclusive hypothesis, the minor premise can also affirm the consequent.

3. The regulative principles and rules for the *categorical syllogism* can be most easily defined if we treat the syllogistic propositions as if they assert a relation between two classes, or the relation of part to whole; in fact, the rules laid down in most text-books are based upon this assumption.

We have seen that in the syllogism, a connection of some sort, established between the middle term and the major term in one premise, and between the middle term and the minor in the other premise, makes the conclusion of the syllogism necessary. Our problem, therefore, is to determine what this connection is, and how we can be certain when this connection between the terms has been established.

Now, if we assume that the syllogistic inference is based upon the class relation, the principle on which that inference proceeds is that of inclusion in, or exclusion from, a class. Thus, to take the classic syllogism: —

All men are mortal;
Caius is a man;
Therefore, Caius is mortal.

Caius is included in the class man, and this class is included in the class mortal beings; hence, Caius also must be included in that class.

Logicians distinguish two sets of rules for the categorical syllogism: general rules, those which apply to any syllogism; and special rules, those which apply to the syllogism in each of the four figures. We shall follow this distinction; and, accordingly, let us first ascertain the general rules and the reasons for them. One such rule is, that the middle term must be universal in one premise at least. The reason for this rule is, that were this term taken only in part of its extension in both premises, it could establish no certain connection between the major and minor terms. A glance at the syllogism will make this evident. Suppose the premises to be the following: —

Some A is B;
All C is A.

Let us quantify the propositions, and the premises become, "Some of A is some part of B; All of C is some part of A." Now it is obvious, that with these premises no connection is necessarily established between C and B; for, although C is in A, it is not necessarily in that part of A which is included in, or subsumed in B. Let us test this rule by a syllogism in a different figure, say Fig. II.

All A is some part of B;
All C is some part of B;

In this syllogism the middle term *B* is not taken in its full extension; and the consequence is it cannot be determined whether *A* and *C* are connected or not; since they can both be a part of *B*, without either being a part of the other. Two circles can be put within the same circle without necessarily intersecting each other. The student who is so minded can test this rule in the other two figures of the syllogism; and he will find the use of the circles as in quantification a serviceable method. We can formulate this first general rule of the categorical syllogism in this way, One premise at least must be universal.

The second general rule of the syllogism is, One premise at least must be affirmative. The reason for this rule is, that if both premises are negative, the relation between the middle term and the other two terms being one of exclusion, no certain connection can be established between those terms. It is just as when two circles are outside the same circle; it cannot be determined in that case whether those circles are outside of each other or whether they intersect.

The third general rule in the syllogism is, that if one premise is particular the conclusion can only be particular. Let the premises be:—

All *A* is *B*; or All *A* is some part of *B*;
Some *C* is *A*; Some *C* is some part of *A*.

It is obvious that with these premises, we can only be certain of the part of *C* that is included in the middle term *A*. Let the premises be:—

All A is some part of B;
Some C is not any part of B.

It can only be certain in this case that some C is no part of A.

The fourth and last general rule is, If one premise is negative, the conclusion is negative. Thus, if the premises are —

All A is B;
No C is B;

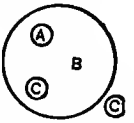
it is obvious that, since C is excluded from B, in which A is included, the relation between C and A is one of exclusion, and that is a negative relation.

Now, it might be supposed that any syllogism which conformed to these general rules would be valid; but examination will show that such is not the case. Take, for instance, these premises: —

All A is B;
No C is A.

These premises do not violate any general rule of the syllogism. Why not, then, draw the conclusion that No C is B? The reason will be at once plain if we note what the minor premise does; it excludes the class C from the class A; and the major premise having included this A class in the class B, we cannot be certain that C is also included in B or is excluded from B.

The following arrangement of the circles shows this ambiguous position of *C*:—



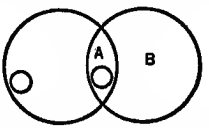
The circle *C* can be in the *B* circle, or outside that circle; and in either position be disconnected from the *A* circle.

Now, observe that these premises from which no conclusion follows are the premises of the syllogism in Fig. I; and that it is the minor premise that is negative. Hence, one condition of a valid syllogism in Fig. I is, that the minor premise must be affirmative.

Let us next inspect the following premises, also in Fig. I.

Some *A* is *B*;
All *C* is *A*.

Why not conclude, Some *C* is *B*? The reason evidently is, that the major premise being a particular proposition, only a part of the middle term *A* is in the major term *B*; and, although the minor premise includes *C* in *A*, it does not necessarily include it in that part of *A* which is in *B*, as the circles will show.



In this diagram the position of the circle *C* is ambiguous. Hence, a second special condition of a valid

syllogism in Fig. I is, that the major premise must be universal. If we unite these two special conditions, we get as the first of the special rules of the syllogism the following; In Fig. I the major premise must be universal, and the minor premise must be affirmative.

We will next examine the premises of a syllogism in Fig. II.

All *A* is *B*;
All *C* is *B*.

Why should not these premises give a conclusion as these propositions would do in Fig. I? The obvious reason is, that these premises violate the general rule which requires that the middle term should be taken in its fullest extension in one premise at least. Now, if we quantify these propositions, it will be clearly seen that the middle term *B* is not taken in its full extension. The use of the circles will make this fact apparent. Two circles can be placed within the same circle without necessarily intersecting. Hence, the second special rule of the syllogism is, In Fig. II one premise must be negative.

We observe also that the syllogism in Fig. II has this peculiarity, that it gives only negative conclusions.

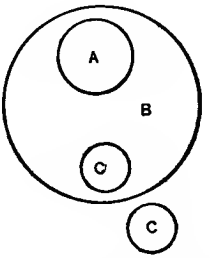
We will next see what special rule, if any, is required for Fig. III. Take the following premises: —

A is *B*;
A is not *C*.

Will these premises give a conclusion? Quantified, they read:—

All of A is some part of B;
None of A is any part of C.

Representing these premises by circles, we get the following:—



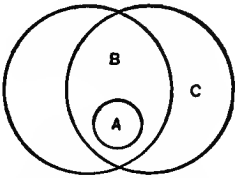
From this it appears that the circle C can occupy either of two positions and remain outside the circle A. Notice it is the minor premise that is negative, and yields this ambiguous result. Hence, we conclude that in Fig. III as in Fig. I the minor premise cannot be negative.

Take again, in the same figure, these premises:—

All A is some part of B;
All A is some part of C.

Since both these premises are universal propositions, we might expect that the conclusion would be universal also, as it certainly is in the other figures we have examined; but examination will show that only a particular conclusion is admissible with these premises. Again,

let the circles be drawn and the truth of this statement becomes clear.



The circle *A* is within two circles and these circles need have only a part of their areas in common. Putting together these two results, we get the third special rule of the syllogism: In Fig. III the minor premise must be affirmative, and the conclusion only can be particular.

Finally, there remains to be examined the syllogism in Fig. IV. This figure, as we have seen, differs from Fig. I only in having the minor premise first. Now, examination of the syllogism in this figure shows that the minor premise cannot be negative. But it also appears that in Fig. IV the major premise cannot be particular. We get the following rule, which applies to syllogisms in Fig. IV: The minor premise cannot be negative and the major premise cannot be particular.

These special rules, we have ascertained and proved, can be reduced to three; since some of them apply to more than one of the figures.

Accordingly, if we add to the general rules for the syllogism the special rules, we can formulate them as follows: —

General Rules. —

1. One premise at least must be universal.

2. One premise at least must be affirmative.

3. If one premise is particular, the conclusion must be particular.

4. If one premise is negative, the conclusion must be negative.

Special Rules. —

1. In Figs. I, III, and IV the minor premise must be affirmative, and in Figs. I and IV the major premise must be universal.

2. In Fig. II one premise must be negative.

3. In Fig. III the conclusion must be particular.

There remains one form of the irregular syllogism for which special rules are required. It is the sorites. Examination of the Aristotelian sorites shows that the chain is broken, if any premise excepting the last is negative; also, that no premise excepting the first can be particular. Hence, for this sorites two rules are to be observed: 1. Only one premise, and that the last, can be negative. 2. Only one premise, and that the first, can be particular. Inspection of the Goclenian sorites makes it evident that it is only the first premise that can be negative, and the last premise only can be particular. Accordingly, the two rules for this sorites are: 1. Only one premise, and that the first, can be negative. 2. Only one premise, and that the last, can be particular.

CHAPTER IX

FALLACIES IN DEDUCTIVE REASONING

SECTION 24

DESCRIPTION OF FALLACIES

A FALLACY is an error in reasoning or inference. It consists in the violation of some principle or condition of valid inference.

There are two principal sources of fallacies: —

(1) Misapprehension of the terms of the premises, or of the evidence supplied by the premises.

(2) A misapprehension of the principles and conditions of right inference.

Fallacies are therefore divided into two main classes, according to the source and nature of error committed, *Material* and *Formal* fallacies. In the material fallacies the error lies in the subject-matter, in some confusion of meaning in special terms or in a misapprehension of the meaning and evidential force of the premises. Hence, the name material to designate this class of fallacies. In formal fallacies, the error is committed in the process of inference, in proceeding from the premises to the conclusion. Accordingly, we can

say, material fallacies give us the wrong premises from which the inference proceeds; formal fallacies draw the wrong conclusions from the given premises. Two examples will make clearer this distinction. Here is a fallacious argument: "A college graduate is sure of the appointment; I am a college graduate; and therefore I am sure of the appointment."

Now, the error in this reasoning lies in overlooking a difference in the meaning of the middle term, *a college graduate.*

In the first premise, it is a college graduate who has those particular qualifications or attributes which will insure his obtaining the appointment. In the second premise, a college graduate is a man who need have only the attributes which every other college graduate has, and in consequence of which I am a college graduate. Therefore, that which makes me a college graduate does not necessarily make me the particular species of college graduate that will secure this appointment.

The next example gives, we shall see, a different kind of fallacy. "Those who think this man is innocent, think he should not be punished; you think he should not be punished; therefore, you think he is innocent." In this reasoning the fallacy lies somewhere on the way between the premises and the conclusion; and a little inspection will find it. The argument is a syllogism in Fig. II, and both its premises are affirmative; and this violates the special rule which requires that, in this figure, one premise shall be negative. This fal-

lacy is therefore formal, while the preceding fallacy is material.

1. **Material Fallacies.** — Having defined fallacies, and explained the chief distinction between them, I shall now describe the fallacies of the first class, the material fallacies. These fallacies, we have seen, consist either in a wrong interpretation of the terms, or of the propositions employed. The description of these fallacies will be more easily followed if we carefully examine typical specimens of them. As the first case, (1) take the following: "All presuming persons are contemptible; this man is therefore contemptible, because he presumes that his opinion is correct."

The error in this argument has its root in a double meaning of the term *presume;* in the first premise it has a meaning to which is attached contemptibleness; in the second premise it does not have that meaning. The fallacy consists in overlooking this difference of meaning, or in assuming an identity of meaning where it is not. Giving to this term its proper meaning in each proposition, it is evident that these two propositions are not so connected that any conclusion can be drawn from them; in other words, they are not real premises, but two propositions which have nothing to do with each other.

(2) The next case for examination is the following: "Pine wood is good lumber; matches are pine wood; therefore, matches are good lumber." In this argument there is no ambiguity in the middle term, as was the case in the preceding argument; and yet there is

a confusion of distinct things. If we examine the middle term in each premise, we shall discover the nature of this confusion. The assertion in the first premise is, pine wood possesses those properties or attributes which are identical with the properties essential to good lumber; the assertion in the second premise is, that matches possess those same attributes or marks which constitute that kind of substance called pine wood. Now this argument assumes that the attributes which make matches pine wood are identical with the attributes which make pine wood good lumber; or to express it in technical terms, the argument confounds generic marks with marks which are not generic, and which, not being essential to the connotation of the name pine wood, are accidents. The marks which make matches pine wood are generic marks; the marks by virtue of which pine wood is good lumber are not its generic marks, and therefore not the marks it has in common with matches; these marks are accidental to its connotation. Therefore the two propositions on which this argument is based establish no connection between pine wood and good lumber. That which makes matches pine wood does not necessarily make it good-pine-wood-lumber.

(3) The next case to be examined admits of two explanations; it is the following: "The holder of a ticket in a lottery is certain to draw the prize; and, since I hold a ticket, I am certain of drawing the prize." One way of explaining this fallacy is to make it a case of simple ambiguity of the middle term, *a ticket;* the term

meaning in the first premise a certain ticket, and in the second premise it means any one ticket. The other interpretation, and, in my judgment, the correct one, makes the fallacy one of the same species as the one just described, a fallacy consisting in the confusion of essential with accidental marks. The argument wrongly assumes that the marks which make me a ticket holder make me a prize-drawing-ticket-holder, which is not necessarily the case.

(4) The fallacy in the next argument, though one of confusion, is of a slightly different sort from those already described. "The Germans are beer drinkers; and since Hans is a German, I infer he is a beer drinker." The technical name of this fallacy is, arguing from a general rule to a special case; and this term well describes the fallacy. The particular source of this fallacious inference is a confusion of a general statement with a universal statement. The universal permits no exceptions; the general allows a considerable number of exceptions. Some one might maintain that the fallacy in this argument belongs to the class of formal fallacies; that it is a case of a syllogism in Fig. I having a particular major premise. The first proposition means some Germans are beer drinkers; hence the syllogism becomes, "Some Germans are beer drinkers: Hans is a German; therefore Hans is a beer drinker," a conclusion which, according to the rule for syllogisms in Fig. I, is inadmissible. It is better, however, to regard this argument as a case of material fallacy, since the error is primarily one of misinterpretation.

(5) Another variety of confusion is presented in the following argument: "All the trees in the park make a dense shade; that pine tree is a tree in the park; therefore it makes a dense shade." The error in this argument consists in overlooking the difference between the trees being all taken together, as they are in the first premise, and the trees considered individually, or distributively, as they are so considered in the second premise. The technical name of this fallacy is the fallacy of composition.

Were I to argue that since no one of the trees in the park makes a dense shade, there is no dense shade in the park, I should commit the converse fallacy, that of division, which consists in overlooking the difference between considering the individuals of a class separately and these individuals taken together.

(6) The next case to be examined presents a material fallacy, but a fallacy of quite a different character from those we have examined. Suppose the following argument is presented: "It must be on the whole a good thing to allow every man an unbounded freedom of speech, because it is highly advantageous to the community that each individual should enjoy a liberty, perfectly unlimited, of expressing his sentiments."

The fallacy in this argument does not consist in a confusion of things that are different, but in the failure to perceive an identity where it exists, in this case the identity in meaning between the conclusion and a premise, or that which is to establish the conclusion; in other words, the argument begs the question; it

moves in a circle. The source of this error is a misconception of that which is needed in order to establish the given conclusion; hence, something is offered as proof which is the thing to be proved.

Let us take another example of this kind of fallacy. "That doctrine should be condemned because it is heresy." One who does not accept this argument could not accuse the reasoner of begging the question in the same sense in which the preceding argument is called begging the question. Certainly, this reasoning is not open to the criticism that it moves in a circle; the only reply to this argument is that it makes use of an unwarranted premise, that it assumes without right the truth of the proposition, "All heresy should be condemned." Hence, begging the question, or *petitio principii*, as it is designated, is done in two ways: either by taking as proof of the conclusion that which is virtually the conclusion itself; or, by taking for a premise a proposition the truth of which needs first to be proved.

Another technical designation for the first species of *petitio principii* is *circulus in probando*. Of these two forms which this fallacy assumes, it is not difficult to recognize and establish the first; but it is by no means the same with the second form. When is the fallacy of unwarranted assumption committed? I think no definite or absolute answer can be made to this question. A reasoner is always liable to the charge of making unwarranted assumptions. There are, however, three ways by which one can safeguard his rea-

soning from this attack: the reasoner must confine his premises to self-evident propositions; or he must be at pains to establish each proposition that he is to use for premises; or he must get the acceptance of the premises at the outset.

(7) The last case of fallacious reasoning illustrates another species of the fallacy due to misapprehending the premises. "The more correct the logic, the more certainly will the conclusion be wrong, if the premises are false; therefore where the premises are wholly uncertain, the best logician is the least safe guide."

In this argument the proposition, "The best logician is the least safe guide," is substituted for or mistaken for the proposition, "the best logician will draw no certain conclusion where the premises are uncertain." This last proposition is the only admissible conclusion from these premises. The fallacy consists in assuming that two propositions are identical, which are really different. This fallacy, technically called *ignoratio elenchi*, is closely allied to that of *petitio principii;* however, this distinction can be made between them: in *petitio principii* there is ignorance of that which constitutes proof of a given proposition; in *ignoratio elenchi* there is ignorance of that which is to be proved. In *petitio principii* wrong propositions are used to establish the conclusion; in *ignoratio elenchi* some other proposition is taken for the conclusion.

The following are other designations of this fallacy: arguing to the wrong point, irrelevant conclusion, *argumentum ad hominem*, *argumentum ad populum*, etc.

The two last expressions designate varieties of this fallacy; one being some retort, or attack upon the reasoner, instead of an answer to his argument; the other being the substitution of appeal to the prejudices or passions for a refutation of the argument presented.

Ignoratio elenchi may occur in serious reasoning; but it is more commonly, however, the expedient of the hard-pressed opponent, the disingenuous advocate, or public debater who, instead of meeting an argument, seek to divert the minds of their hearers from the issue. The reply of a barrister to a request from his attorney in a certain case illustrates the character of this fallacy: "We have no case; abuse the plaintiff's attorney." As a further illustration of the *ad hominem* variety of this fallacy, we have the following: A member of the legislature is advocating the passage of a certain bill; his opponent, instead of showing that the proposed measure is not desirable, offers as argument the inconsistency of the member in now advocating a measure which only a short time ago he opposed.

Again, suppose one is arguing that a certain measure should become a law, and the reply is, "This is a bad measure, for it is supported by bad men; see what sort of men are in favor of this law; you will be in fine company in supporting such a measure!" Here, we have an example of the *ad populum* variety of the *ignoratio eclenchi.*

2. **Formal Fallacies.** — These fallacies consist, as we have seen, in a violation of the rules for valid inference. Some of these fallacies occur in the simple form

of deductive reasoning called immediate inference; they arise from misapprehending the principles of obversion, conversion, contraposition, added determinants, and the relations of opposition. Thus, it is not uncommon for the student to confound obversion with a proposition that resembles the obverted one, but is wholly different in character.

If, for the obverse of the proposition, "All *A* is *B*," one gives, "What is not *A* is not *B*," he commits the fallacy of wrong obversion, this proposition being by no means the same as, "No *A* is not *B*."

So with other forms of immediate inference; each is exposed to a fallacy. The following proposition was once given to a class in an English university, "A stitch in time saves nine," and of a large class but few gave the right converse, most merely giving the proposition with the verb in the passive voice; Thus, "Nine stitches are saved by a stitch in time."

Again, the *A* proposition is not infrequently converted without limiting the extension of the new subject term. It is by no means needless to caution students against erroneous inferences based upon the various relations of opposition. The contrary relation is a snare to more than a few incautious reasoners; more than half the students in a class in logic have inferred the truth of one contrary from the untruth of the other contrary proposition, and to quite as large a proportion of the class it seemed correct to assert that both subcontraries can be false.

Coming now to the fallacies in mediate or syllogistic

inference, we shall see that they all consist in a violation of the conditions of valid syllogisms. We will first note those which occur in hypothetical syllogisms. The only fallacies to which this form of reasoning is liable are the fallacy of affirming a consequent and that of denying a condition or antecedent.

It is the categorical syllogism that furnishes most of the formal fallacies, and these we will now examine. We shall find that these fallacies consist either in a non-inclusion of a term where such inclusion is necessary to the inference, or in the failure to perceive that a term is not taken in its full extension, or in the failure to observe the quantity and quality of the premises. Accordingly, formal fallacies in the categorical syllogisms fall into these classes: —

(1) fallacies of nonsubsumption,

(2) fallacies of undistributed middle term,

(3) fallacies of wrong quantity and quality.

Nonsubsumption may affect either the middle or the major term. A syllogism in Fig. I with a negative minor affords an example of nonsubsumption in the middle term; thus, "All A is B; no C is A; therefore, no C is D, is a false syllogism, and the fallacy in it is technically the fallacy of nonsubsumption in the minor term. Nonsubsumption in the middle term occurs in syllogisms with two negative premises.

As an example of the fallacy of undistributed middle term take a syllogism in Fig. II, with two affirmative premises: —

All A is B;
All C is B;
Therefore all C is A.

The fallacy here consists in not observing that the middle term B is not taken in its full extension.

The fallacies of the third class have no technical designations. Cases of them are the following:—

(1) whenever a universal conclusion is drawn from premises in which one proposition is particular,

(2) when a particular conclusion is drawn where a universal is possible,

(3) when an affirmative conclusion is drawn from premises containing one negative proposition.

There are no accepted technical designations of the fallacies in this last group. It would describe them well did we call those under—

(1) fallacies of proving too much; those under

(2) fallacies of proving too little; and those under

(3) fallacies of a proposition of the wrong quality.

Section 25

Classification and Technical Designation of Fallacies

The various kinds of fallacies in deductive inference have been described and incidentally to most of them have been given their technical designations. To some extent a classification has also been given. I shall not follow the usual practice, and attempt to give a com-

plete classification. It is not, I think, possible to do so, for the reason that some of the fallacies, especially some of the material fallacies, cannot be successfully classed. They do not belong decisively and indisputably to any one of the classes into which they are put by this or that logician.

Nor is the matter of correct designation so simple as it would seem. However, it may be advantageous to the student to have the suggestion of a plan or scheme of classification which he can carry out, or reconstruct in the interest of a more satisfactory classification.

I will outline such a plan, and then add some observations upon the principles of the classification I suggest and also upon the technical designations of certain fallacies.

The fallacies of deductive inference are of two kinds; they fall into two main groups, one group comprising the formal fallacies, the other group comprising the material fallacies. The formal fallacies again subdivide into fallacies of immediate inference and fallacies of mediate or syllogistic inference. The fallacies of immediate inference can be further separated into fallacies of equipollence and fallacies of opposition. The fallacies of equipollence include the following: —

1. False obversion.
2. False conversion.
3. False contraposition.
4. False added determinants.

The fallacies of opposition comprise the following: —

1. Fallacy contrary.
2. Fallacy contradictory.
3. Fallacy of subcontrary.
4. Fallacy of the subalterns.

The fallacies of mediate inference fall into the following subdivisions: —

1. Fallacy of nonsubsumption.
2. Fallacy of nondistribution of middle term.
3. Fallacy of wrong quality and quantity in the conclusion.
4. Fallacy of wrong minor premise in hypothetical syllogism.

The material fallacies fall most conveniently into the following subdivisions: —

1. Fallacy of confusion.
2. Fallacy of mistaken proof or evidence.

The confusion fallacies again subdivide into: —

1. Simple ambiguity.
2. Composition and division.
3. Accident.

The fallacy of mistaken proof subdivides into: —

1. The fallacy of *petitio principii* or begging the question.
2. Fallacy of *ignoratio elenchi* or irrelevant conclusion.

The fallacy of *petitio principii* presents two varieties, unwarranted assumption and circle in proof, *circulus in probando.*

It is hardly worth while to specify the varieties of the *ignoratio elenchi* fallacy.

I add a few words in explanation of some things in this scheme of classification. Accident fallacies are those which give most trouble, and those about which there is disagreement among logicians. The term should be applied to those fallacies and to those only in which the error consists (as I have shown in the analysis of these fallacies), in the confusion of generic or essential marks with accident, or nonessential marks. This fallacy does not consist, as Jevons appears to think, in arguing from a general rule to a special case, and the converse, arguing from a special case to a general rule. These two fallacies are species of the accident fallacy, the technical names for them being *a dicto simpliciter ad dictum secundum quid* and *a dicto secundum quid ad dictum simpliciter.*

The fallacy of composition and division is by some logicians regarded as a species of accident fallacy, but incorrectly. The source of this fallacy is the confusion of a class composed of individuals that are taken together with a class the members of which are taken distributively. The fallacy is based upon the assumption that one can always predicate the same thing of the individuals of a class taken separately that one can predicate of them taken together, and conversely, one can predicate of things taken together what one can predicate of them taken separately.

The exact designation of the fallacy which includes *petitio principii* and *ignoratio elenchi* is not easy to find. I think, however, it is more accurately described by the terms I have used than by those in common

use. The essence of this fallacy is misconception of what the premises prove, and what sort of premises are needed to prove the thing we seek to establish. Hence, this misconception takes two forms, — either a misconception of what is requisite for the proof sought, or a misconception of what is to be proved, and therefore of what the given premises prove. In the former case we have *petitio principii*, in the latter, we have *ignoratio elenchi*.

Section 26

THE VALUE OF THE SYLLOGISM

Regarding deductive reasoning, and the syllogism in particular, opposite views are held. Some maintain that the syllogism is a useless survival of mediæval logic; that it is no legitimate form of inference, being a mere *petitio principii*. Others have maintained that the syllogism is indispensable to valid reasoning, being the only form in which inference can be expressed if it is to be clearly valid. The truth lies between these extreme views. The syllogism has important uses, and serves purposes of no inconsiderable value in the discipline of thought. But it is true, at the same time, that this form of inference has rather narrow limits. It is not applicable to all cases of genuine inference. Some of these cases cannot without awkward and unnatural constructions be made to take the syllogistic form; other cases either do not come under the forms of the syllogism, or plainly violate its canons.

In discussing the function and value of the syllogism, I shall first consider its limitations and its defects; and secondly, I shall show in what consists the value of this much-decried instrument in reasoning.

The syllogism has a limited range of utility. It is useless and embarrassing in some cases of deductive reasoning. Such are inferences based upon relations of space, time, and quantity. Here are some instances: given the position of *A* at the right of *B*, and the position of *C* at the right of *A*, with this datum we at once draw the inference that *C* is at the right of *B*. Now, let us put this reasoning into the syllogistic form, and we shall get some such construction as the following: "What is at the right of *A* is also at the right of that position in space of which *A* is at the right, which in this case is *B*; *C* is at the right of *A*; therefore *C* is also at the right of *B*."

Such a syllogism is certainly a clumsy and unnatural construction, and quite needless, since the property of space which is the foundation of all such inferences is directly perceived, and once perceived such a roundabout way of reaching a conclusion is useless.

Take as another instance, "*A* follows *B* in time, *C* follows *A*." These premises give at once the conclusion, *C* follows *B*, the foundation of inferences of this sort being clearly the law of time sequence. Now, express this inference in the traditional syllogism and this rather awkward structure is the result, "Whatever follows *A* in time, follows also that which *A* follows, which in this case would be *B*; *C* follows *A*; therefore *C* follows

B." One more instance: "*A* is greater than *B*; *C* is greater than *A*; therefore *C* is greater than *B*."

A glance at these propositions, while it assures us that the third is the conclusion from something contained in the two preceding propositions, shows us that these propositions do not constitute the syllogism of formal logic, for there is no major premise, or rather the major premise is implied and not expressed. Now, supplying this premise, we get the following: "Whatever is greater than a given quantity, say *B*, is greater than that quantity than which this quantity is greater; *C* is greater than this given quantity *A*; therefore *C* is greater than *B*." It can hardly be maintained that such a form of reasoning facilitates the passage of thought from datum to conclusion. The direct perception of the relation of quantity is the bridge over which thought passes easily and surely to the conclusion in such cases.

But not only is the syllogism useless and a hindrance in certain cases of reasoning, there are cases of genuine inference which the syllogism does not recognize, which fall outside of its forms of admissible inference. Here are a few instances of such inferences:

(1) "Horses are vertebrates; dogs are vertebrates." As the student at once perceives, these are the premises of a syllogism in Fig. II, and no conclusion is admissible according to the rule for valid syllogisms in this figure. From these premises nothing can be determined respecting the relation between horses and dogs. But is it after all true that this datum yields no significant and rational belief?

Let us suppose that horses and dogs are names for a large number of individuals which, on the basis of common properties, have been, for convenience, put into these two classes. Let me further suppose I have observed that all the individuals in these two classes have a number of properties in common with a third class of individuals called vertebrates, or that both these classes can be put into this larger group; now, does not the fact that dogs and horses possess a number of properties in common justify me in believing in some degree that they have other properties in common; or, in other words, that they agree in other respects, so that the one class may be included in the other?

This is the sort of reasoning on which we proceed in matters of practical interest, and which has a recognized value in science. I conclude from the datum, in this case, that horses and dogs probably agree in other attributes, in addition to those which make them both vertebrates. My reasoning is to this effect: if these two classes of animals agree in these respects, they probably agree in other respects.

This conclusion is a probability; but the syllogism does not recognize mere probabilities. But probability is a rational belief, and the inference that leads to such a proposition is not less logical in character than is the inference which the syllogism sanctions. Probability is the guide of life, and what guides life must be rational.

(2) One more illustration of a genuine inference, which is, according to the syllogistic canons, illegitimate.

I observe that a number of particular substances, *A*, *B*, *C*, *D*, etc., exhibit a certain mode of behavior, say a chemical reaction of some sort, and I draw the inference from these particular facts that all substances like these will exhibit the same reaction. Now, such an inference is forbidden by the rules of the syllogism, or rather, the syllogism does not recognize such an inference. From such particular premises no conclusion can be drawn according to the conditions of a vallid syllogism. But here again my inductive inference, giving a probable conclusion, is as logical a process as the processes which are valid according to the syllogism. This generalization from experience, like the inference from analogy, is the expedient of daily life, and as rational a way of dealing with the matters of our experience as any of the methods of formal inference. Thus is it shown that there is a large class of genuine and useful inferences which lie outside the field of the syllogism.

But not only is it true that many cases of valid inferences are not recognized by the syllogism, there are also cases of valid inference which it is claimed by some logicians violate the syllogism. Paradoxical as it sounds, a valid inference is possible from two negative premises. Take as an example of such a syllogism the following: —

X is not a knave;
X is not a fool;

therefore, he who is not a knave is not necessarily a fool; or knavery and folly do not in all cases go together;

or because a man is not a knave, do not conclude he is a fool. Now, here is a syllogism in Fig. III with two negative premises, which, according to the rules of the syllogism, cannot give a conclusion. I will leave it to the student's judgment to decide whether there is or is not a conclusion from these premises.

Let us change these premises so as to get a valid syllogism:—

X is a knave;
X is a fool;

therefore, some knaves are some fools, or knavery and folly sometimes go together; and where you find one you may find the other. Let the student compare these two conclusions, and say which of them is the more significant or suggestive, the one from the affirmative, or the one from the negative, premises.

Let us add one more case which brings out more distinctly the legitimacy and the value of conclusions from negative premises. "None of the men in the town *A* are rich; none of these men are unhappy; therefore, riches are not necessary to happiness; people can be happy who are not rich. Do not, therefore, conclude that a man who is not rich is not happy."

Here is a conclusion from negative premises that is not only admissible, but quite as important as would be the conclusion were one or both of these premises affirmative. Defenders of the traditional syllogism have maintained that these alleged cases of valid conclusions from two negative premises are not really

violations of the syllogism; because there are not, as is alleged, two negations in the premises, one at least of the premises, it is held, is really affirmative.

This defence is valid against some of the instances which have been given by critics of the syllogism; but it does not avail against such cases as the first given above. The premises there are as genuine negations as can be found, and a significant conclusion follows. A better defence of the validity of the rule which forbids a conclusion from negative premises is to observe the limits within which it is valid, the exact nature of the relation between things which the syllogism contemplates.

Turning again to our first example, let us interpret the two propositions according to the relation on which the traditional syllogism is based. Now, there is one sense in which these premises do not permit a conclusion, or rather there is one sort of conclusion that does not certainly follow from these premises. We cannot from these two negative propositions reach a definite conclusion respecting the relation between these two classes, fools and knaves; we cannot determine whether some knaves are or are not in the class of fools. It is just this relation between two classes that the categorical syllogism contemplates, and consequently in such a case as the one given no certain conclusion follows. Another fact must be kept in mind, the only conclusions that the syllogism recognizes are those that are certain; the syllogism knows nothing of probability. When these two facts are borne in mind, the contradiction of

the syllogism which some logicians see in such cases is apparent rather than real. These cases of conclusions from negative premises, like the cases we have noted above, do not fall within the field of syllogistic inference, as that inference is defined in formal logic.

So much for the limitations of the syllogism. Now let us turn to the positive side, to the uses of the syllogism.

There is one thing which in all reasoning is of primary importance, — definite premises. There must not be vagueness and obscurity here, if there is to be any clear and definite issue from the premises. One value of the syllogism is that it enables and compels the reasoner to make the first step in argumentation definite and exact. The syllogism is an instrument by the aid of which the exact meaning and scope of the propositions which form the datum can be determined. For this purpose, no better instrument has been devised than the syllogism. The syllogistic analysis and coupling of propositions is the most effective means yet devised by which the premises are defined and made perfectly clear.

Again, in the process of inference, the syllogism is the most serviceable instrument for keeping the way clear from premises to conclusion. The principles and rules of syllogistic inference are guideboards which read so plainly that only the heedless or very stupid reasoner can miss his way, so thoroughly is he safeguarded against misleading ways. No more simple or efficient instrument has been discovered for detecting errors

into which our own thinking may fall, and errors in which the sophistry of another reasoner may try to entangle us.

This discipline of thought which the use of the syllogism yields has been too lightly appreciated. The ability to go at once and unerringly from a given proposition to all that is implied in that proposition, and from two propositions to all that follows from their admission, is no small or easy acquisition; and it is an abundant justification of the syllogism and a sufficient reason for its retention in the training of the intellect that it gives this ability, without which one is not a good reasoner.

PART TWO

THE LOGIC OF SCIENCE

CHAPTER X

INTRODUCTORY

THE division of our study in logic is based upon the twofold aim in thinking, consistency and truth of fact. The customary title Inductive Logic is not, it seems to me, a fortunate one; first, because it implies that there are two kinds of logic, each with principles of its own; whereas logic and logical principles are of one and the same nature, whatever may be the subject-matters to which they are applied. The fundamental purpose of logic is to ascertain and apply the principles which are regulative for our thinking. Now, whatever may be the special aim of this thinking, this main function of logic is the same.

The title Inductive Logic is not fortunate for another reason: the term *inductive* has two meanings, and taken in one meaning, it is too narrow to define this division of logic; taken in the other meaning, it requires an extension of the term which ought not to be given

it. Induction means an inference which proceeds from particular facts of observation. Induction in the usage of some logicians also means those various processes by which science explains the facts of nature, and which are employed in all investigation. To make this term cover all these processes and methods by which scientific knowledge is attained is to extend the meaning of the term beyond its proper limits. The title I have given this division of logic marks distinctly the aim proposed, — an exposition of the principles of logical thinking which are employed in science.

Section 27

The Meaning of Science

A successful execution of the task now undertaken requires that we have at the outset the right conception of science, — its subject-matter, its aim, and its limits. I shall briefly discuss these before proceeding to the exposition of the logic of science.

1. **The Facts of Physical Science.** — The objects of scientific knowledge are phenomena and phenomena only. Phenomena are things which are perceived or which can under supposable conditions be perceived; they are events which occur; they are changes which take place, or processes, such as motion, which go on and can be observed and be matters of exact measurement and description. The world from the point of view of science and for the aim of science is a phenomenal world.

2. **The Province of Scientific Explanation.** — Scientific explanation consists in finding for these facts and events the most general laws to which they conform. A phenomenon is scientifically explained when it is shown to be an instance of a general law, applicable to all phenomena of like description; or when this phenomenon is referred to some definite antecedent condition, which, being given, this phenomenon invariably follows.

For the right understanding of scientific explanation, two terms must be accurately defined. One is the term *cause*, the other the term *law*. For the purpose of science, a cause need only be an antecedent phenomenon on which a given phenomenon invariably depends. Invariable antecedence in time is the only necessary mark of causal connection. The term *law* in science means a uniform and invariable order in which phenomena occur.

Laws of nature are statements of the uniformities of succession and existence among phenomena, and the ideal of science is the reduction of these uniformities to the fewest in number and the simplest in character. Laws are, therefore, not things which exist or have any meaning apart from phenomena; they are only descriptive formulæ by the aid of which science describes in the simplest and most comprehensive terms the manner in which the phenomena of the world occur. Laws do not prescribe how things shall take place; they are formulæ for describing how things *do* take place.

3. **The Limits of Scientific Explanation.**—Science is limited in two respects: first, in respect to the subject-matter of its explanation; and secondly, in respect to the explanation it gives. There are some things which science presupposes as the necessary condition of its explanations. It presupposes, for instance, the uniformity of nature. This is the working postulate of science; without it not a step can be taken. But this principle on which science depends is not something which science has discovered; it is the mind's trust in the rational character of the world and the adaptation of the world to our purposes and needs; it is an essentially ethical faith that Nature will not disappoint our expectations, nor put us to intellectual confusion in our attempts to know and practically to control our world.

Science presupposes such things as matter, force, space, time, etc. The exact meaning of these conceptions lies outside the field of science. It is the function of science to describe in the simplest and fewest possible terms the motions of that which we call matter; but science does not undertake to say what matter is. Science explains the phenomena of life, the evolution of living beings; it describes their various behaviors; but it does not tell us what life is, whence it comes or whither it goes. Science describes the various functions of mind, mental phenomena; it formulates the laws in accordance with which they occur; it investigates the various connections between these phenomena and phenomena of the physical order. Science

traces the evolution of mind from its simplest discernible manifestation to its highest and most complex functions; but science leaves unanswered the question, what is the mind; what is the thinking, feeling, and willing being called ego, mind, soul, self.

Nor is science a final or complete explanation; it stops short of the goal of rational explanation. There are two questions which man as a thinking being necessarily asks about everything, — the question of whence and how, and the question of why, what for. It lies within the province of science to answer all questions of genesis, all questions of how. It does not lie within her province to answer the other more significant and often more urgent questions.

The function of science, we have seen, is description. So far as the world is a describable world it belongs to science. But there is more than a world of description. There is also a world of valuation; there are meanings and values of which science can take no account. At this boundary line between the answer to the questions whence and how, and the questions why and what for, science submits to the dictate, Thus far and no farther canst thou come.

4. **The Special Problems of Science.** — Having explained the nature of science and delimited its field, I will next explain the special problems that belong to science, and in a general way explain the methods by which these problems are solved. The first of these problems is the ascertainment of causal connection between known phenomena; this connection is between

phenomena that are observed or can be made observable by experiment.

This problem belongs to the first stage of scientific explanation; and it arises out of the character of our experience, the way in which the world is directly given to our minds in simple sense perception; and this problem means the reconstruction of this rather chaotic world of direct experience, so far as to reduce its events and phenomena to some degree of uniformity of occurrence. The search for causal connection is the attempt of rational thought to get behind mere appearances to the real world.

The world of our immediate or direct experience is very unlike the world which science constructs or discovers. Order, unity, causal connection do not lie upon the surface; they are not immediately presented to our senses; they must be sought for and constructed out of the data which our sense perceptions supply. "The order of nature," says Mr. Mill, "as perceived at first glance presents at every instant a chaos followed by another chaos. We must decompose each chaos into the single facts; we must learn to see in the chaotic antecedent a multitude of distinct antecedents. . . . The regularity which exists in nature is a web composed of distinct threads, and only to be understood by tracing each of these threads separately."

These first threads which science traces out are those of causal connection in the observable parts of this web; and what renders this problem difficult is the fact that the causal connections are, to quote a statement from

another writer, "embedded in a mass of extraneous and irrelevant material from which it is our business to dissect them out."

The second special problem of science is explanation by hypotheses.

Science cannot stop with the first stage of explanation, if, indeed, that step can be called explanation at all, and not a mere preliminary step to explanation. All real explanation involves a step from the known to the unknown. Now this step is hypothesis, the essence of which is the supposition of the existence of something not seen, not yet known. An hypothesis is an ideal construction. By it thought goes beyond sense and conceives some reality beyond the limits of observation and experience.

The justification of taking this step is that assumption which underlies all our knowledge of nature and all science, the continuity and orderliness of the world. Science assumes that the yet unobserved facts of the world are so related to known facts of experience, are so continuous with these facts present in experience, that both admit of ultimate description in terms of a common formula. And hence an hypothesis is an instrument, a device, for this more comprehensive and accurate description of the phenomena and processes of the world.

The great hypothesis of Newton was thus an ideal construction by means of which, not only the motions of particular bodies, but of every body in space, can be described, and those motions predicted for any

future time. So with the ether hypothesis; it is a grand fiction of the scientific imagination which is most serviceable in reducing to fewer and simpler processes a great number and variety of physical processes.

It must be remembered, however, that hypothesis building is no work of fancy, but a task of serious thought. An hypothesis is no mere flight of the imagination, but a venture of reason and at the bidding of rational thought.

The third special problem of science is occasioned by those phenomena which, on account of their complexity and the obscure conditions on which they depend, do not admit of scientific explanation in the more exact meaning of that term; yet, because these phenomena do present certain uniformities in their occurrence, and admit to some extent of measurement and calculation, they come within the province of scientific method.

These phenomena are of two sorts and, accordingly, two distinct methods are applied to them; these methods are technically known as calculation of chances, and the method of statistics. The former method is applied to those events which, taken singly, have no known cause, but which show a tendency to uniformity of occurrence and admit of calculation with varying degrees of probability. The method of statistics is employed in dealing with those phenomena, which, taken in very considerable numbers or in masses, and observed for considerable periods of time, present certain uniformities and certain persistent characteristics.

The calculations employed in this second method are not applied to the individuals which compose these aggregates, but to the aggregates only, considered in their mass character.

Such are the special problems of science, and such in general the methods by which science effects or seeks to effect the solution of these problems. We pass now to the exposition of these problems and scientific methods in detail.

CHAPTER XI

THE ASCERTAINMENT OF CAUSAL CONNECTION BY OBSERVATION AND EXPERIMENT

SECTION 28

OBSERVATION AND EXPERIMENT

WE have seen that order, unity, and causal connection are not presented to our immediate experience; that Nature presents, instead, a web of tangled and interwoven threads, which present to direct perception a bewildering complexity.

It is the first task of science to trace out these separate threads of causal connection, to disentangle them from the mass of connections which are not causal. Observation and experiment are the instruments we employ in the execution of this task. Accordingly these two operations must have our first consideration.

1. **Observation.** — Experience teaches us that it is no easy thing to observe rightly and successfully, and yet observation is fundamental to all scientific knowledge. Carelessness, inaccuracy, or confusion here vitiate all the results that are gained by this first step, and that must furnish the data for the subsequent stages in

investigation. The difficulties of successful observation and the errors to which it is exposed are chiefly the following: —

(1) The complexity of the phenomena themselves, the fact that every phenomenon we would observe and distinguish is embedded in a mass of coexisting phenomena; it is just one fact in a very complicated setting of incidents, a thread interwoven with countless other threads. Hence the difficulty of isolating the phenomenon we are trying to study, and the difficulty of eliminating the causal connection which this phenomenon sustains to some other phenomenon, from other concomitant conditions with which this phenomenon is not casually connected.

(2) A second difficulty observation encounters arises from the limited time to which direct observation is confined. This time span, owing to the constitution of our mind, is very limited, and the difficulty this fact occasions is aggravated by the circumstance that in this brief time period a number of phenomena are occurring or existing simultaneously.

(3) But successful observation is difficult for a third reason; successful observation depends upon the control and persistence of attention; and attention, unless disciplined by an energetic will, is easily distracted, and liable to be occupied with the nonessential concomitants of the phenomenon we are investigating. Add to the distracting influence of the multiplicity of simultaneously occurring incidents, the influence of our prepossessions, our subjective biases of various sorts, the

tendency to see what we are thus prepared to see, to observe what prior experience and habits dispose us to observe, — add these influences, and the difficulty of observation, arising from wrong attention, can be fully appreciated.

It is owing to these difficulties I have pointed out that observation falls into various errors, the more common of which are the following: —

(1) mal-observation; this consists either in overlooking some important circumstance, or in a wrong perception of the circumstances in which a phenomenon occurs;

(2) confusion of perception with something inferred from what is perceived. This form of error is exceedingly common, and one of the most subtle forms of wrong observation. How difficult it is to keep actual perception distinct from inference any one can appreciate who will introspect a little, or attend carefully to the relations of the same event by different people, equally well informed and equally conscientious observers. One who is familiar with the proceedings of the courts is forced to confess that it is not easy for the most honest person to tell the whole truth and nothing but the truth.

The Requisites of Good Observation. — To be a good observer three things are especially requisite: (1) accuracy and carefulness in perception, (2) power of sustained attention, (3) a good memory. This last requisite may seem to have nothing to do with observation, which is confined to what is present; but, so

narrow, so evanescent is our present perception that no one can observe a present fact and know in any degree what that fact is, who does not remember something at the same time. Part of every phenomenon we are trying to observe has slipped away into the past before we have really observed it; hence, our observation must have in it a constituent of memory. To hold completely and steadily in our grasp the immediate past is thus indispensable to an accurate and complete observation of anything that is occupying our present thought.

2. **Experiment.** — Observation alone, even were it ideally perfect, is inadequate to the task of analyzing the situations in which phenomena occur, and of ascertaining in these situations what phenomena are causally connected. Hence, observation needs to be supplemented by experiment, which is an artifice for enlarging and making observation more exact. Experiment is the instrument of science; it is an artificial treatment of phenomena, an intervention of our agency in the course of events, a subjecting of Nature to methods and tests of our own devising, in order to see more clearly what is the actual behavior of Nature herself.

I will now point out some ways in which experiment aids and supplements observation.

(1) By making possible repeated instances of the same phenomena. Did we need to rely on observation alone we could learn little of some phenomena, because they are of infrequent occurrence; but if we can by

experiment get a repetition of the same phenomenon, we are greatly aided in our observation of it.

(2) By enabling the observer to isolate the phenomenon under observation. We have seen that one difficulty which observation encounters is the complexity of the conditions in which a particular phenomenon occurs. Experiment overcomes this difficulty by isolating this phenomenon; this it does either by eliminating circumstances that are not causally connected with the phenomenon under investigation, or by producing different situations in which the same phenomenon occurs.

(3) By the use of instruments. Here lies one of the great achievements of modern science, the employment of instruments for the measurement and calculation of the events and processes of nature. It is to this use of instruments that modern science largely owes her advance upon ancient science. It is mainly this employment of instruments that has made possible the accuracy and extension of observation on which this great advance of science depends. It is to the balance, the telescope, the microscope, the marvelous apparatus with which modern research is equipped, that we are chiefly indebted for the discoveries and expansion of knowledge which distinguish the past century from those which preceded it.

Modern physical science is based upon the application of mathematics to the phenomena of nature; and instruments are methods of bringing these principles of mathematics into fruitful application to nature.

We shall not be likely to overestimate these advantages of experiment over unaided observation in gaining a knowledge of our world, especially when we reflect upon this attitude to the universe which characterizes experiment, in contrast with the attitude that characterizes mere observation. In experiment man is no longer a passive observer, waiting for facts to be presented to him; he actively intervenes in the course of events; he tries his universe, questions it, and predetermines the sort of answers Nature will give to his questions, by selecting the questions he will ask. Man has found that it is the will of Nature that he that "asketh receiveth," he that "seeketh findeth," to him that "knocketh it shall be opened." Man has learned by experiment that Nature is plastic to his action; that she opens her mysteries to the importunities of experiment; that her word of assurance to him is "prove me and see if I will not reward thee."

Section 29

THE REGULATIVE PRINCIPLES OF OBSERVATION AND EXPERIMENT. THE SO-CALLED INDUCTIVE METHODS

The exact problem for observation and experiment is to ascertain which of the antecedents, or concomitants, of a given phenomenon is its causal antecedent, or is one of its causal antecedents.

The solution of this problem consists in the analysis of the situation in which the given phenomenon occurs; and this analysis means separation between the con-

comitants of a phenomenon that are non-causal and the concomitants which *are* causal.

The process is thus one of elimination, elimination of non-causal circumstances from the totality of conditions in which the phenomenon under investigation occurs. Now, the inductive methods (as they are rather unfortunately named) are simply the ways in which this analysis and this elimination are effected; hence, it would be more appropriate to call them methods of analysis, or methods of elimination; for that is precisely their function.

These methods in their present formulation we owe to John Stuart Mill, although Mill did not discover them; some of them were recognized by Bacon, and they were more fully recognized by Sir John Herschel. Nor must it be supposed that these methods were invented, that the principles they formulate are *a priori*. Mr. Mill did not invent any of the canons or rules he formulated; nor did he borrow them from other logicians; he learned from the practice of men in different departments of science, the methods they followed in their investigations and reasoning; and these methods of induction and their canons are only the formulation of the actual procedure and the accepted principles which men in science have always followed.

I shall now present these methods substantially as they are formulated by Mill.

1. **The Method of Agreement.** — This consists in observing the instances of the phenomenon under investigation, and noting in what single circum-

stance all these instances agree, while they differ in all the other material circumstances; or, noting what single circumstance is always present, and the only one that is always present, when the given phenomenon occurs. Mill's canon for this method is, "If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause or the effect of the given phenomenon."

2. **The Method of Difference.** — According to this method there is a comparison of the instances in which a given phenomenon occurs, with the instances in which this phenomenon does not occur; and it is the sole circumstance in which these instances differ that is noted. The following is Mill's canon for this method: "If an instance in which the phenomenon under investigation occurs and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon."

3. **The Joint Method or Method of Double Agreement.** — The distinctive feature of this method is the double employment of the method of agreement, this method being employed both in the instances in which the phenomenon occurs, and in the instances in which it does not occur. The method thus affords two distinct proofs, each proceeding independently of the other, and each corroborating the other. The follow-

ing is the canon for this method: "If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ, is the effect, or the cause, or an indispensable part of the cause, of the given phenomenon."

4. **The Method of Residues.** — This method is employed in those cases in which some of the concomitants of the phenomenon are already known to be causal antecedents and consequents; and the method consists in subtracting these from the totality of concomitant circumstances, so as to leave as the residuum, the causal antecedents yet to be ascertained. The canon of this method is, "Subduct from any phenomenon under investigation such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedent."

5. **The Method of Concomitant Variation.** — This method consists in ascertaining what variation in a given phenomenon occurs when a definite variation occurs in some other phenomenon. Its canon is, "Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some effect of causation."

The student will better understand the use of these methods if I add a few examples of their employment.

Elimination by Agreement. — I observe after taking a particular kind of food, I am invariably ill; a careful comparison of all instances in which this result follows shows that the taking of this kind of food is the only material circumstance in which they all agree; I infer from this fact that it is this kind of food that is the cause, or at least in part the cause, of my being made ill.

Elimination by Difference. — A man, known to be in good health at a certain moment of time, falls dead; examination discovers that a bullet has penetrated his brain. A mass of gunpowder is in a magazine, a lighted match is put in contact with it, an explosion follows. The sole differencing circumstance in the instances of the man in health, and the man dead, was the bullet in his brain. Likewise in the two instances, that of gunpowder in the magazine, and gunpowder destroyed by explosion, the sole differencing circumstance was the lighted match in contact with the powder. We say the bullet killed the man, and the match caused the explosion.

Elimination by Double Agreement. — As an example of the employment of this method, I take the following from Fowler's "Inductive Logic" (p. 163). A ray of light proceeding from incandescent hydrogen is passed through a prism, and it is invariably found that, in the spectrum thus obtained, there are two bright lines occupying precisely the same position; moreover, rays of white light proceeding from various incandescent substances are passed through incandescent hydrogen and the emergent light is then broken up by a prism.

In the spectra thus obtained, it is found that there are invariably two dark (or under certain circumstances two bright) lines occupying exactly the same position in the spectrum. If we try the same experiments with any other elements than incandescent hydrogen, although we may obtain bright and dark lines, we never find these lines occupying the same position in the spectrum as the two lines in question.

As this case is not so simple as the ones given in illustration of the two first methods, I will analyze it. First, by the method of simple agreement it is shown that the ray passing through incandescent hydrogen and the invariable position of certain lines in the spectrum are causally connected things, since this passing through incandescent hydrogen is the sole agreeing circumstance in the instances in which the phenomenon occurs. Secondly, it is shown by the same method applied to the negative instances that the absence of a ray passing through incandescent hydrogen is the sole antecedent on which the non-occurrence of this phenomenon is observed.

Elimination by Residues. — The classic illustration of this method is the discovery of the planet Neptune. The facts are briefly these: Certain perturbations in the planet Uranus had been observed since 1804. It was known what amount of perturbation in the motions of this planet was due to the influence of known heavenly bodies. Deducting the effects of the known influence of these other bodies, there remained the perturbations for which a cause was to be discovered; and as the

student probably knows, Mr. Adams in England and M. Le Verrier in France almost simultaneously calculated the position of some planetary body which could occasion these disturbances in the motions of Uranus. Dr. Gill of the Royal Academy of Berlin turned his telescope to that region of the heavens, and discovered the planet Neptune.

Elimination by Concomitant Variation. — A good example of this method are some observations upon the grip epidemic in New York, made by Weather Forecaster Dunn. Mr. Dunn came to the conclusion, that humidity with change in temperature was the most important element in causing the spread of the disease. The facts on which this inference was based are the following: (1) The fatality was most marked when the humidity was at its maximum, and there was a sudden fall of the temperature. (2) The higher the humidity and the more sudden the fall of temperature, the greater was the number of deaths. (3) When, on the other hand, the temperature and the humidity dropped at the same time, there was a decrease in the death rate.

A comparison between these methods may serve to bring out more closely their distinctive features. These methods, as has been shown, have a common function, that of eliminating the non-causally connected concomitants of the given phenomenon. Each of these methods effects this elimination in a different way. In most of the instances in which the causal antecedent is sought, more than one of these methods can be employed;

and, when this is the case, the evidence of causal connection is of course materially strengthened. Let us first compare the methods of agreement and difference.

A first point of difference between these methods is the principle on which each method proceeds. The method of agreement goes on the principle, that whatever circumstance can be eliminated without affecting a given phenomenon, is not causally connected with this phenomenon; the principle of the method of difference is, whatever circumstance *cannot* be eliminated without affecting the given phenomenon, is a cause of this phenomenon.

A second difference between these methods concerns the character of the instances with which each deals, and the way in which these instances are treated. The method of agreement requires us to observe or obtain by experiment instances which agree in but a single circumstance; the method of difference requires instances which agree in all the circumstances but one. Thus it is the agreeing circumstance that is important in the one method, while the differing circumstance is the important one in the other method; hence the names that aptly distinguish these methods. Again, notice that in the method of agreement comparison is made between all the instances in which the phenomenon occurs; in the method of difference the comparison is made between an instance in which the phenomenon occurs with an instance in which this phenomenon does not occur. Only the *presence* of the phenomenon

is ascertained by the one method; both its *presence* and *absence* are ascertained by the other method.

Elimination by Agreement and Difference. — This method differs from the method of agreement only in the circumstance that it takes account of negative as well as positive instances of the given phenomenon; that is, instances in which the phenomenon does not occur as well as those in which that phenomenon occurs.

A comparison of this third method with the method of difference is not unimportant, because the student is liable to confound these methods, or at least to suppose that the method partakes of double agreement of the distinctive character of the method of difference. The name joint method is not so good a term as the other term, double agreement; because it implies this mistaken connection between the two methods. It is true that the method of double agreement has this feature in common with the method of difference, viz. in it two sets of instances are observed, instances in which the phenomenon occurs, and instances in which it does not occur; but there is no further agreement between them; on the contrary, there are these differences: —

(1) In the method of double agreement it is the agreeing circumstance that is noted in both sets of instances; in the method of difference, it is the disagreeing circumstance that is noted.

(2) In the method of double agreement instances in which the phenomenon occurs are compared with each other, and instances in which the phenomenon does not

occur are compared with each other; in the method of difference the comparison is between instances in which the phenomenon does not occur, and instances in which it does occur.

The method of residues when compared with the other methods presents these two peculiarities:—

(1) It does not of itself establish a causal connection. It only eliminates known causally connected concomitants of a given phenomenon. The residual phenomenon with its concomitants is a problem to be solved either by the other methods, or in some instances by the method of hypothesis, as will be shown later. Thus, in the discovery of the planet Neptune all that was accomplished by this method was the separation of the given phenomenon and its concomitants into two parts, one containing antecedents and consequents known to be causally connected, the other containing a phenomenon and an unknown or unobserved cause; and it was by hypothesis and verification that this causal antecedent was discovered. The case would not have been materially different had this cause been among the observed concomitants of the given phenomenon; it would have been by the use of one of the other methods that this cause was discovered.

(2) The second peculiarity of the method of residues is, that it is used to ascertain causal connection, not only between observed phenomena, but between an observed phenomenon and something that is not observed; while the other methods are limited to causal connections between observed phenomena.

Section 30

THE LOGICAL VALUE OF THE METHODS OF OBSERVATION AND EXPERIMENT

I shall discuss in this section the evidence of causal connection which these methods afford. Let us first assume that the situations which these methods presuppose are actual and realizable; we will assume that these methods can be ideally carried out, that such instances as the method of agreement, for instance, contemplates are met with in experience.

Even under such ideal conditions as we have supposed, these methods come short of satisfying the canons of formal logic. It is quite certain that if *A* is a cause of a given phenomenon, *A* will always be present when that phenomenon occurs; but, to infer from the uniform presence of *A* as a circumstance that is always present that it is a cause, is to commit the fallacy of affirming a consequent. Judged, therefore, by the canons of formal logic, these methods do not make us logically certain of causal connection. But if the evidence possible by these methods falls short of certainty, it can and does approximate that ideal of evidence. Between the probability of causal connection which these methods, even under actual conditions, attain and certainty there is no difference of any practical value. We attain to a conviction that is so practically sufficient, and so rationally satisfying, that no really sound mind feels an inclination to doubt, or could justify itself in so doing.

There is, however, great inequality in these methods in respect to their evidential value. The weakest of them is the method of agreement; and the strongest, the method of difference. Perhaps next in evidential value should be placed the method of concomitant variations. These methods, as we have observed, can very considerably corroborate each other; since in most cases more than one of them can be employed. The method of agreement is relatively weak for the reason, as Mill observes, that it at best only establishes the presence of a particular circumstance when a given phenomenon occurs; it cannot make us certain that other circumstances are not also present but unobserved; nor can it make us certain that if this particular circumstance were not present, the phenomenon would not occur. Let us suppose that *A* is always present when *B* occurs, and in fulfillment of the requirements of this method, *A* is the only observed circumstance that is always present when *B* occurs; this situation permits no less than four inferences:

(1) *A* and *B* are related as cause and effect.

(2) Both *A* and *B* are effects of some unobserved cause.

(3) *A*, though always present, is not itself the cause of *B*, but the cause of some circumstance, either observed or unobserved, which is causally connected with *B*.

(4) *A*, though the cause of *B*, is not the sole cause; some other circumstance present would be the cause of *B* in the absence of *A*, this other circumstance being latent owing to the influence of *A*.

Thus our first method comes short of establishing causal connection. The method of difference is far more cogent in the inference it warrants. Mill accords to this method the highest degree of evidence, amounting to practical demonstration even under actual conditions. "It is by the method of difference alone that we can ever, in the way of direct experience, arrive with certainty at causes" ("Logic," p. 282). The reason for the greater cogency of this method, Mill finds in the fact that the nature of the combinations which it requires is much more strictly defined than in the method of agreement.

This method, requiring that the two instances be alike in all circumstances save one, and also involving both presence and absence of the given phenomenon, is a much more effective instrument for the elimination of non-causal concomitants than is any one of the other methods. But what most of all makes this method the strongest of the methods is the fact that it permits a completer employment of experiment; it is preëminently an experimental method, and therein lies its effectiveness. It is thus possible to introduce into a given state of circumstances a change that is of a perfectly definite nature, and to observe what results. To quote again from Mill ("Logic," p. 281:) "We choose a previous state of things with which we are well acquainted; so that no unforeseen alteration in that state is likely to pass unobserved; and into this we introduce, as rapidly as possible, the phenomenon which we wish to study; so that in general we are

entitled to feel complete assurance, that the preëxisting state and the state which we have produced differ in nothing except the presence or the absence of this phenomenon." "If a bird is taken from a cage, and plunged into carbonic acid gas, the experimenter may be fully assured that no circumstance capable of causing suffocation has supervened in the interim except the change from immersion in the atmosphere to immersion in carbonic acid gas."

To a considerable degree what is true of this last method is true of the method of concomitant variation; this likewise admits of the use of instruments of exact measurement; and where such instruments can be employed, it is possible to establish relations that are so definite in character as to make the inference of causal connection scarcely less compelling than is the evidence afforded by the method of difference. Since it is upon the definite character of the variations that this method relies, its evidential value is proportionate to the degree of definiteness that these variations present. Now, when it is possible to establish mathematical relations, such as ratios, relations of weight, volume, intensity, motions, etc., the evidence of causal connection thus afforded, it will be readily perceived, is very strong.

The method of double agreement, owing to the negative instances it considers, has greater evidential force than does the method of single agreement; but, inasmuch as it does not involve a comparison between positive and negative instances, and does not make

possible the use of experiment to such an extent as the method of difference, this method is distinctly weaker than the former.

In this estimation of the logical value of these methods we have assumed that they are employed under conditions that completely satisfy their requirements. Experience teaches, however, that such conditions are in no cases afforded us. It is this discrepancy between the hypothetical conditions of these methods and the actual conditions to which they are applied that constitutes the inherent weakness of them all; though this weakness affects some of them to a greater degree than it does others. The possibility of there being unobserved concomitants of the given phenomenon despite our most careful analysis; the possibility that more than one circumstance stands in causal connection with the given phenomenon; the possibility that the invariable coexistence of the two phenomena in our rather limited experience may be a non-causal coincidence only,—these possibilities, it must be confessed, weaken the evidence of causal connection which in very many cases it is possible for us to obtain.

The complexity of nature is too great, and our powers of analysis and accurate observation are too limited, to enable us to attain more than a reasonable probability that we have discovered causal connection between phenomena, that we have successfully traced out the numerous and intricate threads of causal linkage that compose the vast web of nature. Still more is this true of the phenomena of human actions,

— social phenomena. To discover, in this realm, laws of causal connection is a goal of endeavor yet far in advance of any present achievement. Observation here being our main reliance, and the method of agreement the only one that in many cases is admissible, we can understand how precarious are the conclusions in most reasonings in the so-called moral sciences.

CHAPTER XII

EXPLANATION BY HYPOTHESIS

ALL real explanation in science proceeds by hypothesis and verification. The ascertainment of causal connection by the methods just described is only a preliminary step toward explanation. The causal connections themselves only give the phenomena to be explained that orderly character, that form in which they can become the data for scientific explanation. These causal connections themselves become new problems for explanation.

Hypothesis is the great instrument of science. Every important advance in man's knowledge of the universe has involved this step from the known to the unknown. Nothing is so characteristic of the great minds in science, as the ability and the courage to make hypotheses and rigorously to test them. It is for this reason that scarcely any quality is more requisite to the investigator, the discoverer, than what Tyndall happily calls the scientific imagination. Original men have possessed this faculty in a high degree. To this rôle of the imagination in science, we owe all the great discoveries, the brilliant achievements, and the most successful working hypotheses that have distinguished the great

century now passed. Hypothesis has been already defined, and its function in a general way explained. I shall now proceed to an exposition of the use of hypotheses in science.

SECTION 31

THE ESSENTIAL FEATURES OF A SCIENTIFIC HYPOTHESIS

The Requisites of a Legitimate Hypothesis. — Because hypothesis is a step beyond the known, beyond the solid ground of experienced facts; and because this step is a venture in which imagination carries us in one sense to worlds unknown, it would be very erroneous to infer that any sort of step, any kind of venture or flight of imagination, is permitted in science, provided in some way we can get back again to our actual world. Fictions are permitted in science, but only such fictions as help to the understanding of facts, only such fictions as enable us to link facts into an orderly, a coherent and rational universe. Hence, the requisites of a permissible hypothesis are: —

(1) That it conforms to the analogies of experience. By this I mean that whatever agent or mode of action of an already known agent is supposed, it must not be so unlike that which we already know in experience that it cannot be clearly conceived in terms of our experience. Knowledge involves a step to the unknown; but this step cannot be an absolute break in the continuity of thought and possible experience, or we are left as

ignorant as we were before taking this step. There must be some term of relation between that which the hypothesis supposes and the datum from which it starts; and no term of relation is possible between what is absolutely unknown and our known world. Science, therefore, permits the construction of no hypotheses which involve the conception of something totally unlike that which we already know.

(2) The second requisite of a legitimate hypothesis is the possibility of deducing from it phenomena of experience; and this deduction must be based upon relations that are rational. Any other hypothesis violates a fundamental condition, viz. that it shall explain, that it shall lead back to the known. A merely supposed something from which we can get nothing in the way of rendering given phenomena more intelligible than they are already, merely mocks us with the semblance of explanation. The necessary assumption on which science proceeds is the essential, the rational, continuity of that which is not yet known, with the facts of experience.

An hypothesis is a thought construction by the aid of which we make this continuity definite and sensibly realizable; its function is thus to extend our world in terms of possible experience. Hence, no hypothesis is permissible that sets up objects which are unrelated to the objects of actual experience; and from which, consequently, we cannot deduce the objects and processes of which the world of experience consists.

The scientific imagination, it must be remembered, is imagination working under the control of reason and

for the ends of knowledge. Science permits this voyage upon unknown seas and to unknown lands; but only if it be no idle or aimless venture, but a serious quest of truth, a voyage of possible discovery for the purpose of enlarging our knowledge and our practical control of nature.

Section 32

The Method of Explanation by Hypothesis

Four steps have been distinguished in this method:

(1) Constructing the datum;

(2) Constructing the hypothesis;

(3) Deducing the consequences from the supposition;

(4) Comparing these consequences with facts of experience.

Induction, hypothesis, deduction, and verification are the technical names for these four steps. The first of these processes, however, does not properly belong to explanation, because explanation presupposes the datum already defined or construed; and this construing or definition of the datum belongs to the stage in science we have already described, — that of observation and experiment. The problem must first be accurately stated before a solution is undertaken; and hypothesis is just a method of solving a given problem.

Nor is it advisable so to distinguish deduction and verification as to make these processes separate steps; these are only distinguishable elements in one process, which is verification, or proof of the hypothesis.

Accordingly, there are two and only two stages or steps in explanation by hypothesis; these are: (1) the construction of the hypothesis, and (2) its verification or proof. We shall now discuss these processes.

1. The situations in which we construct hypotheses are various; they range from the simplest facts of observation which daily life presents, to those situations in which, by the regulated methods of observation and experiment, order and causal connection have been established among the more complex phenomena of experience. Whatever be the situation or the character of the phenomena that confront us, the essential operation of constructing an hypothesis is the same; it is a conjecture, a conception of some agent or mode of action, by means of which the new facts can be assimilated to what we already know, can be fitted into a coherent and mentally satisfying experience. Intellectual perplexity in the presence of given phenomena, uneasiness and dissatisfaction until this perplexity is removed by rational explanation, are the impelling motives to all hypothesis building.

To the formation of a good and serviceable hypothesis, two things are requisite, — accurate knowledge of the given facts, and analogical suggestion; by the latter I mean the detection of significant agreements between given facts and other facts already explained. I shall have occasion in another place to discuss the nature and value of analogical inference; here, I will only remark that its scientific value lies chiefly in its suggestion of hypothesis. Indeed, analogical suggestion

is the basis on which every hypothesis rests; we might say the invention of an hypothesis is an analogical inference.

2. The second step in explanation by hypothesis is verification, which consists of a statement of what phenomena ought to be observed if the hypothesis is true, and a comparison between these phenomena and those of actual experience. In this deduction of consequences and their comparison with actual facts consists the test of the hypothesis. These two moments in the process of verification can be expressed as the premises of a hypothetical syllogism; the major premise is the statement of the hypothesis and that which is deduced therefrom, and the minor premise states the result of the comparison with experience, either by asserting that there is, or asserting that there is not, such agreement between the hypothesis and the facts of experience. When the phenomena deduced from the supposition agree with experience, the hypothesis is said to be verified. If this agreement is complete, so that there are no facts left unexplained, the hypothesis is said to be completely verified, sometimes said to be true or proved. Incompleteness of verification, it is obvious, is a matter of degree; the partial verification may leave considerable areas of fact unexplained, and yet be an admissible and serviceable hypothesis.

Two distinctions are of sufficient importance to deserve attention at this point. They are: —

(1) The distinction between complete verification and complete proof, and

(2) Between incomplete verification and disproof of an hypothesis.

It is necessary to the complete proof of an hypothesis that it should be completely verified; but a completely verified hypothesis is not thereby completely proved. An hypothesis is completely proved only when that which was supposed, is otherwise discovered to be a known fact of experience, or when that hypothesis is demonstrated to be the only one which can explain the given phenomena. To be completely proved, then, an hypothesis must either cease to be mere hypothesis and pass into fact, or, remaining an hypothesis, be the only possible one in the given situation.

An instance of a completely proved hypothesis was the discovery of the planet Neptune. Up to the hour when the telescope revealed that body, Neptune was merely a supposed being, a completely verified hypothesis, to be sure, but still an hypothesis only. The revelation of the telescope completely proved that hypothesis, and it did so by converting hypothesis into fact; Neptune became a known fact of experience.

To illustrate the second condition in which an hypothesis is completely proved, let us suppose that the hypothesis was that of a heavenly body so situated in space that no telescope had as yet found it; if, under that condition, this supposed body was the only one which could explain the disturbances in the motions of Uranus, astronomers would have been as certain of its existence as they were of the existence of Neptune after the telescope had brought it into the sensible world.

But a sole hypothesis, such as complete proof presupposes, is an ideal, not an attained actuality. It may be shown that the given hypothesis is the only one yet proposed that explains the phenomenon under investigation; but to demonstrate that no other hypothesis is conceivable or will ever be framed which can explain that phenomenon is something our human minds, subject to change and growth, cannot do.

The most that can be claimed for any hypothesis is that it is so far as known the only one that is admissible; and this approximation toward a sole hypothesis constitutes very strong evidence, stronger, of course, than the evidence afforded by complete verification; but it is evidence that falls short of complete proof. There is, consequently, but one way in which an hypothesis is completely proved; and this absolute proof at the same time transforms the supposed, into known reality.

The other distinction, that between partial verification and disproof, must not be overlooked if we would understand the actual procedure in science. A perfectly admissible and even very serviceable hypothesis may be one which leaves a part of given phenomena unexplained; but no hypothesis is admissible after it is shown to be in contradiction with any one fact of experience. One *contradictory* fact is a disproof of an hypothesis, while *many unexplained* facts are not incompatible with a tenable and useful hypothesis.

These distinctions explained, we pass now to a discussion of the evidence for hypothesis. We begin with

the evidence afforded by complete verification. Judged by the canons of the hypothetical syllogism, such complete verification does not establish the truth of the hypothesis; indeed, to accept such an hypothesis as true commits the fallacy of affirming the consequent. But this discrepancy between the evidence demanded by the hypothetical syllogism and the evidence that is available for hypotheses in science only serves to accentuate that difference we have insisted upon between formal logic and reasonings upon matters of fact. The syllogism, as has been shown, recognizes no conclusions that are not certain, and admits no proof that does not establish such conclusions. In matters of fact, probability is all that the best evidence obtainable can give us. Nor, do the principles which are regulative for syllogistic reasoning afford any standard by which the evidence for scientific hypotheses can be tested and measured.

The grounds on which the scientific mind accepts any particular hypothesis cannot be formulated in the terms of formal logic. In the world of concrete facts other principles are regulative for our thought; and there are other criteria of rational belief than those which belong to the world of mere conceptions. Experience is not only the datum or starting point for all scientific reasoning, but also the test by which the evidential force of all such reasoning is to be measured. To work completely in experience, to make experience intelligible, coherent, and practically, as well as theoretically, satisfactory, is the criterion of a true concep-

tion or hypothesis. Does an hypothesis enable us to comprehend the given phenomena, does it enable the mind to forecast them, does it enable us to know what to expect and what practically to prepare for, so that our mental prevision and our actions as well are defined and made sure? These are the tests by which an hypothesis is judged; and these afford the standard by which the evidence for it is measured.

Next, therefore, in credibility to the sole hypothesis, we should ordinarily rank the completely verified hypothesis; while hypotheses that are only partially verified would be weakest in point of evidence. But there is one circumstance which affects the relative evidential value of the completely verified and the incompletely verified hypotheses, — it is the character of the phenomena they explain. There are conditions under which an hypothesis which is only partially verified is accepted with more confidence than a completely verified hypothesis is under other conditions. An hypothesis may be completely verified within a relatively narrow range of phenomena, and with phenomena of simple character, which is not entitled to so much confidence as another hypothesis which is incompletely verified, but is applicable to a very wide range of phenomena, or to a very complex and peculiar phenomenon. Accordingly, the evidential value of an hypothesis is, in part, determined by the extent or unique character of the phenomena to which it is applied.

Summing up this discussion upon the evidential value of hypotheses, we can formulate the chief points as follows: —

(1) Next to a completely proved hypothesis the one entitled to most confidence is the hypothesis which, within the limits of what is known, is the only one that can explain the given phenomenon.
(2) Other things being equal, the completely verified hypothesis ranks next in credibility.
(3) Other things being equal, that hypothesis is most credible which is applicable to the widest range of phenomena, or to phenomena most complex in character.
(4) An hypothesis is admissible, provided it explains any part of the given phenomenon and is contradicted by no phenomenon.
(5) An hypothesis is no longer tenable when a single phenomenon is found which contradicts this hypothesis.

Section 33

THE VALUE OF REJECTED HYPOTHESES

It is not alone the true hypotheses which are useful to science; science is also indebted to her rejected hypotheses; each one of these has done something to prepare the way for the more successful ones that have taken their places. The observation, often made, that the pathway of science is strewn with the wrecks of exploded theories, discarded hypotheses, involves an erroneous conception of the nature and growth of our human knowledge. Truth is not reached by a single bound. Seldom has one leap of the scientific imagina-

tion brought the mind to the right, or at least to the final, explanation.

Hypotheses in science become established as do species in the organic kingdom, — through the struggle for existence and the survival of the fittest. But the unsuccessful and the defeated competitors have contributed to that progress which has left them behind to perish. Hardly has there been an hypothesis so erroneous as not to contain some element of truth, and which has not in consequence of that truth helped the establishment of a truer one. It is by the testing and rejection of the bad hypotheses that the conditions become better defined, the problem more correctly stated, and the requisites of the true hypotheses more accurately apprehended.

The pathway of all our knowledge leads through errors and partial failures. Science has won her progress through half truths and rejected untruths; she has advanced not alone by successful steps, but by mistaken and corrected steps. The men who have been most successful in science are those who have most generously recognized the measure of truth in conceptions they have rejected, and who have taken most pains to understand doctrines which the progress of science has made no longer tenable. The truth is the whole; and the sure mark of the conception that can claim final truth is its capacity to fulfill, and not to destroy, the conceptions it displaces.

CHAPTER XIII

THE THIRD SPECIAL PROBLEM IN THE LOGIC OF SCIENCE

SECTION 34

THE CALCULATION OF CHANCES

As we have seen, the phenomena which give rise to this third problem are of two sorts: —

(1) Individual events whose occurrence can be predicted with varying degrees of probability; and

(2) Phenomena which, considered as aggregates or masses, present such uniformities as to admit of prediction with a high degree of probability.

We shall now consider the phenomena of the first description.

The terms *chance* and *probability* do not signify a quality of any phenomena or events in themselves considered. There are no chance or probable events of that sort in the real world; such an event would be one which had no reason for its occurrence, and which was consequently without connection with any other phenomena. Such an event is absolutely unthinkable and rationally impossible. These terms describe cer-

tain states of mind or mental attitudes toward certain events in our real world; in other words, chance and probability describe states of our minds and not qualities of any events in nature. They are confessions of ignorance, the limitation of our knowledge.

Did we completely know our world, such mental attitudes would be impossible. But, on the other hand, were we utterly ignorant, there would be neither chance nor probability; it is because we are both knowing and ignorant that we can speak of some things as chance events, and regard their occurrence as probable. A chance event is an event of whose cause we are ignorant. A probable event is an event for the occurrence of which some reasons exist in what we know of it, or of the class to which it belongs.

Probability properly means any conviction that is less than certainty, and a conviction the evidence or reason for which admits of estimation or measurement. When, for example, I say it is probable that the war in the East will soon be brought to an end, I express a conviction of a certain measurable degree. If, in addition, I say the probabilities are four times as great that Japan will come off victorious as are the probabilities that Russia will triumph, I give to my conviction a definite measurement; I express its relative strength in quantitative terms. To be more accurate, I mean by this statement that there exist for my mind four times as many reasons, or four times as strong a reason, for expecting the success of Japan as there are for expecting the success of Russia.

The importance of making clear the exact meaning of chance and probability justifies our lingering longer upon this topic. Let us, accordingly, note the situations in which we can properly characterize a contemplated event as one of chance occurrence. That situation is clearly one in which we must know something — we must know that some event is to occur; but, at the same time, we do not know what particular event, or what is the particular description of the event that is to happen. In other words, we know that one of several possible events is to be actual; but we do not know which *one* of these possible events is to become the actual one.

Let me describe this situation in somewhat different terms. It is a situation in which we know that more than one event is possible. It is a situation in which we know that one event at least will be actual. It is a situation in which we have no better reason for expecting one of the alternative possible events, than for expecting any other one of them, our ignorance of causes being equally distributed among the possible events. Now, that state of mind in which I entertain the occurrence of any one of those possible events is what we should mean by chance. If I regard the strength or degree of conviction with which I expect the occurrence of any particular event, that mental state is what should be meant by probability.

Thus, chance and probability can characterize the same event, — it is a chance event, because its particular cause is not known; it is a probable event, because

its occurrence is expected with a given degree of conviction.

And this leads to our topic, The Calculation of Chances. What is it to calculate chances and what are the methods of such calculation? Our best way of answering these questions is to examine some concrete cases in which this is done. Let us take as the first case, throwing a die. Antecedent to the throw, it is known that one of the six faces must come uppermost; the structure of the die makes this fact certain. We know also that some cause will determine a particular side to be uppermost; but, as we do not know what that cause is, the reasons we have for expecting one side to come uppermost are no greater than are those for expecting any other one of the six sides to be uppermost. We therefore say the chances are equal. Because these chances are equal, and because there are six possible events, the chances that it will be a particular side, say a six-spot, that will be uppermost, are but one fifth as many as the chances that favor any one of the remaining five sides; or the chance of a six-spot coming uppermost is one in six of the total chances. Expressed in other terms, our calculation in this instance is, the chances against my throwing a six-spot are five to one. Now, the analysis of this case shows that a strictly mathematical measurement applies to the reasons there are for expecting a particular event, — the six-spot side coming uppermost.

This mathematical calculation assumes two things: —

(1) That our minds are not affected by any experience of the results from prior throws;

(2) That we are equally ignorant of the special causes which will determine any one of these sides to be uppermost, these reasons being simply counted, and not estimated or weighed.

As a second case, let me suppose a box containing black balls and white balls mixed in a proportion which I do not know. Now, in such a situation, antecedent to my drawing a ball we should say the chances are equal, and the probability that I shall draw a white ball is one to two, or $\frac{1}{2}$. Now, let us suppose the box contains white and back balls, in the proportion of twenty white balls to five black balls; under these conditions, my expectation of drawing, say a black ball, is expressible by the fraction $\frac{5}{25}$ or $\frac{1}{5}$.

In the two cases now examined, we have a strictly mathematical measurement, or calculation of chances. This is so because a definite number of events is considered, and each one of these events, so far as we know, is equally possible; and because no other circumstance influences our judgment.

The next case presents a somewhat different situation. It is again a box containing balls; but now, neither the number nor the color of these balls is known. In this situation it is not possible to calculate chances prior to a result ascertained by drawing. I can entertain no expectation respecting the sort of ball that is to come out at the first draw. Now, suppose I draw six white balls in succession, what

are the chances respecting my next draw? Have I more reason to expect a white ball next time than for expecting some other color? May I presume that the box contains only white balls, and if so, can the degree of this presumption be mathematically estimated?

Before answering these questions, let us note carefully the difference between this case and the two preceding ones; this difference is the circumstance, that in the first two cases the calculation of chances was made prior to and independently of experience; while in this case, it is the result of the successive drawings that forms the basis of a calculation of chances, if such a calculation is admissible. In those cases the influence of experience was precluded; in this case experience is the sole determiner of my state of mind.

Does this experience supply a basis for the same sort of calculation as that we made in the preceding instances? There is no question that this uniform experience of drawing only white balls justifies an expectation that the next draw will give a white ball rather than a ball of any other color; the only question is, Can we give a mathematical expression to this expectancy or probability as we did in the other cases? This question is to be answered in the affirmative and for this reason; — after the six successive drawings of white balls and before the seventh drawing, we have essentially the same situation that the other cases presented. Having drawn six white balls and no balls of another color, I may assume that the white balls are six times as many as the balls of other colors; and, consequently

in the seventh drawing, the chances of a white ball coming out are expressed by the fraction $\frac{6}{7}$; and of a ball that is not white, $\frac{1}{7}$.

Let us suppose that of the six drawings, four have given white balls and two red balls; then, in the seventh draw, the probability of a white ball coming out would be expressed by the fraction $\frac{4}{7}$, and the probability of a red ball by the fraction $\frac{2}{7}$; while the probability of a ball of some other color would be indicated by the fraction $\frac{1}{7}$.

In the calculation of chances we distinguish two sorts of cases: (1) those in which the number of alternative possible events is determined by the known conditions under which these events must occur, — throwing dice, drawing cards from a pack, etc., are instances of this class of cases; and (2) those cases in which the occurrence of a phenomenon a certain number of times, in succession, either without or with interruption, is the basis for calculating the chances of the next occurrence; our third case is an instance of this class.

The calculation of chances we have considered relates to a single occurrence of the same event; the calculation cannot be the same for more than one occurrence of the same event in succession. For instance, the probability of throwing a six-spot twice in succession is not one half as great as the probability of a single occurrence of this sort, but is expressed by the fraction $\frac{1}{36}$, and the probability of getting a six spot three times in succession is $\frac{1}{216}$. Hence, theoretically regarded, there is a rapid diminution of the chances favoring the repeti-

tion of the same event in successive instances. This decrease is not in terms of an arithmetical series, but in terms of a geometrical series. From this exposition, we derive the following rules for the calculation of chances: —

(1) For a single occurrence of the specified event in the first class of cases, the probability is expressed by a fraction having for its numerator, unity, and for its denominator, the number of possible events considered.

(2) For a single occurrence of a specified event of the second class, the probability is expressed by a fraction having for its numerator the total number of times the specified event has occurred, and for the denominator this number increased by one.

(3) For the occurrence of the same event more than once, the probability is expressed by a fraction whose numerator is unity, and whose denominator is the number of possible events raised to the power denoted by the number of times the given event is to occur in succession.

In our exposition of the doctrine of chance thus far, we have assumed that the theoretical calculations accord with the actual results obtained. There is, however, a discrepancy between theory and fact in the so-called chance events; and the problem presented by this discrepancy is to determine the amount of discrepancy that is compatible with the chance character of these events. This problem is better stated, perhaps, by this

question: After how many repetitions of the same event may we infer that there is some special cause operating to produce this event? or, When are we justified in believing that the coincidences are more than should occur, if they were only chance coincidences?

Numerous experiments have shown that the theoretical and the actual results approach agreement as the number of trials is greatly increased. For instance, one hundred throws of a penny gave seventy heads and thirty tails, but in upward of five thousand throws the number of heads closely approximated the theoretical number. Similar results have been obtained from a very protracted series of drawings of cards from a pack. These experiments indicate that the theory of probability holds true, if a sufficiently long run of instances is obtained.

Now, it is just this fact of a discrepancy between theoretical results and actual ones in the limited series, but which tends to disappear as the series is prolonged, that gives significance to our question, — When does this excess of actual over theoretical coincidences justify the belief that some particular cause is operative in producing this result? For instance, after how many heads in succession should I be justified in believing that the penny is one-sided? or after how many six-spots uppermost should I believe that the die is loaded?

This question hardly admits of a definite answer, so much depends upon the character of the phenomena and the conditions of their occurrence. I might be justified in believing that something more than chance

coincidence exists in one case, while in another case the excess of actual over theoretical coincidences would not justify such a belief. With such simple phenomena as tossing a penny, or throwing a die, or drawing a ball from a box, one seems justified in inferring a special cause when the coincidences are much in excess, and when they persist rather than diminish as the experiments are continued. But with more complex phenomena, the conditions of which are obscure, such a belief would not be justified.

As a matter of fact, the theory of probability has little influence upon our beliefs regarding future events. We do not regulate or measure our expectations of particular events, whose causes we do not know, by the rules for calculation of chances. Experience and our knowledge of similar cases determine mainly and properly our beliefs. If I throw a six-spot twice in succession, it does not seem improbable that the next throw will give a six-spot also; but it would seem very improbable that I should get this result five times in succession. The reason for this difference in my expectation is, that experience has shown that two sixes in succession is not uncommon, but a succession of five sixes is very uncommon. Those who are adepts in games of chance regulate their ventures by what they have found to be the habits of these phenomena; what is called a run of luck, or the tide, are these habits which seem to belong to all phenomena.

The calculation of chances, however, is not a useless method, simply because it does not regulate our prac-

tical beliefs; this treatment of phenomena possesses a scientific value; because, by it the way is prepared for investigations that lead to the discovery of causal uniformities and for the use of hypotheses. The calculation of chances is sometimes a first step in bringing a group of phenomena, or a special phenomenon, within the domain of explained facts. The calculation of chances is thus a method of more accurate description of certain phenomena; and this more accurate description is an indispensable preliminary to scientific explanation.

SECTION 35

THE METHOD OF STATISTICS

The second class of unexplained phenomena comprises those events which present certain uniformities when considered in considerable numbers or masses, and when observed through considerable periods of time. It must be carefully noted that these uniformities hold true, not of the individuals that constitute these aggregates, but of the aggregates as such. Thus, when it is said that in a given population, say ten thousand, the mean death rate is $\frac{1}{100}$, 1:100, it is not meant that this uniformity holds true of persons taken singly or individually, but that out of this given aggregate the *mean* or *average number* of deaths is one hundred.

By the mean or average number in this method is meant that number which remains relatively con-

stant during a given period of time; and by the mean or average individual or person is meant that fictitious individual who presents those characteristics which are selected as descriptive of this group. Thus, if I say the average German is five feet and eight inches in height, I do not mean that any one actual individual German is just five feet eight inches in height, but I mean that, could the height of every German be made equal, that height would be five feet eight inches. This mean or average German is therefore a fictitious individual. With these explanations we pass to our topic, The Method of Statistics.

Statistics are any facts which are ascertained for a specific purpose; thus, to gather statistics relative to the effect of a certain occupation upon the health of those who are engaged in it, means that one ascertains such facts as the following: the number of persons engaged in this occupation, their ages, their sex, the localities in which they live, etc. Statistics are thus selected facts, facts of a definite character, and always numerically defined, and always for a purpose previously determined. This method includes two operations: —

(1) Gathering statistics;

(2) The ascertainment of uniformities and mean averages presented by the phenomena thus specifically grouped. For example, statistics relating to accidents by railway travel are gathered. From these as data the average number of accidents for a year is ascertained, and the ratio of this number to the num-

ber of persons who travel during this time. And, finally, it is shown that this mean ratio remains approximately constant during a period of years. Thus a uniformity for a class of phenomena is established.

The value of this method of dealing with phenomena is mainly practical, though its scientific value is not unimportant. The practical service of this method is illustrated by some of the most important business organizations or organizations for social improvement. The great organizations of life and property insurance have their foundation in those uniformities which this method has ascertained, and derive their stability from the constancy of the mean ratios, shown in the actuary tables of these companies. Observation covering a long period of years has shown that the causes, whatever they may be, which produce death, injuries, and the destruction of property, so operate as to produce a mean ratio that is relatively constant within a selected group of persons, or for a given aggregate of property valuations. We have only to recall the use of statistics in education, in social institutions such as hospitals, asylums, prisons, etc., to recognize the wide field within which this method is useful.

Statistics are of indispensable service to the legislator, to the economist, and to the student of social problems. But for this method, uniformities in phenomena so complicated and seemingly without law, and whose causes are so obscure, could never have been discovered. But practical utility is not the only

value which the statistical method may claim. Statistics afford valuable data for science; they suggest new lines of inquiry, and set new problems, the solution of which enlarges the boundaries of science.

It was observed, in discussing the calculation of chances, that the striking deviations from the results deduced in accordance with the theory suggest new hypotheses; and these, verified, add new territory to the domain of science. So with statistics; any marked deviation from the computed average or ratio hitherto maintained constitutes a fresh problem; and the methods of observation, experiment, and hypothesis are brought into use, with the not infrequent result of new laws being discovered, and causes hitherto hidden being brought to light.

Statistics thus give opportunity for sagacious suggestions and fruitful investigations. They are of great aid also in rendering observation more varied and more precise; they give opportunities for experiment, — nay, they are of the nature of experiments; and, finally, they are serviceable in testing hypotheses, by making verification more critical and more complete. I have made use of the investigations of Weather Forecaster Dunn in illustrating the method of concomitant variations, but these investigations afford so good an example of the scientific use of statistics, that I shall borrow from them again. Selecting the period from March 22 to May 16 in the year 1891 (the time of the grippe epidemic in New York City), Mr. Dunn prepared a chart which gave the total

number of deaths from grippe in this period. Other statistics were gathered relating to the weather conditions, when there was the greatest number and the least number of cases, and when the fatality was the greatest. Now, it was by means of such statistics that Mr. Dunn succeeded in establishing, with very great probability, a causal connection between weather conditions — particularly degree of humidity and temperature changes — and the increase and diminution of this disease.

CHAPTER XIV

GENERALIZATION FROM EXPERIENCE AND ANALOGY

SECTION 36

INDUCTIVE GENERALIZATION AND ITS VARIETIES

THIS seems to be an appropriate place to discuss the function and logical value of two closely allied forms of inference we have already explained in an earlier chapter.

1. **Inductive inference**, it was there explained, presents two varieties — one of which I will venture to call inductive generalization; the other is known as analogical inference, or analogy. Inductive generalization must not be confounded with the hypothetical step in explanation; to frame an hypothesis and to draw an inference are distinct things. Inductive generalization is the extension of what has been observed in certain cases to other cases which have not been observed; an hypothesis involves a conception of something which may be very unlike what has been observed. Inductive generalization is not explanation of anything in present experience; hypothesis, as we have seen, *is* a

method of explanation, — it is framed for the purpose of explaining what is given in experience. Inductive generalization has two functions: —

(1) It constitutes a preliminary step to scientific explanation;

(2) It establishes other uniformities or approximate uniformities of experience, for which no explanation is yet found, and which may be destined to remain merely empirical laws.

Generalizations from experience, according to Mr. Mill, are of two sorts: absolute generalizations, and those which are approximate only. Absolute generalizations are derived from experience that has presented no exceptions to the observed uniformity which is the foundation of the inference; approximate generalization is based upon an experience in which, in the great majority of instances, the same thing has occurred, but in which exceptions to this rule have also occurred. Absolute generalizations are expressed in universal propositions; approximate generalizations, in particular propositions. Thus, all crows are black, is an absolute generalization; most men seek their own interests first, is an approximate generalization.

Approximate generalizations are of little scientific value, but they may possess great value for conduct; our conduct in matters of great importance sometimes has no other guidance. This is especially the case with much of our conduct in relation to our fellow-beings. Mr. Mill observes, "All propositions which can be

framed respecting the actions of human beings, are merely approximate; we can (for example) only say that most persons of a particular age, profession, country, rank in society, have such and such qualities; most persons when placed in such and such circumstances act in such and such a way" ("Logic," p. 418).

2. **Analogical Inference.** — The service which analogical inference renders to science has been pointed out in the discussion of hypotheses; it remains to consider the other functions of this mode of inference, and to estimate its logical value. The scientific value of analogy, apart from its suggestiveness in explanation by hypothesis, is slight. This inference is relatively strong only when the resembling properties on which it is based are not merely numerous, but are important; and the difficulty of estimating the character of the resembling properties, instead of merely counting them, renders the use of analogy of doubtful service. But, slight as is the service of this inference to science, its practical value is often great.

For many of our strongest beliefs and those which profoundly influence our actions, analogy is the only evidence we go upon. For example, the conviction we have, that our fellow-beings are men of like passions as ourselves, suffer and enjoy what we do, and recognize the obligations we accept, etc., has no other logical ground than the inference from analogy. The expressive acts and deeds of our social fellows are the basis on which we attribute to them the same thoughts, feelings, and purposes that we express by like

acts and deeds. So is it with our belief in purpose, design, both in the case of productions we attribute to beings like ourselves, and in the case of organic structures in nature. The principle of teleological explanation is analogy.

The fact that analogy is so common a method of forming beliefs, and the fact that for so many of them there is no other justification, make desirable a more explicit statement of the principles which should regulate our use of analogy. These regulative principles are the following: —

(1) The importance of the resembling properties, rather than their number, should determine our belief. Weigh, rather than count, the points of likeness and difference.

(2) The strength of an inference from analogy is proportioned to the reason we have for believing any property of that from which the inference starts is connected with that property to which the inference proceeds; a single property which we have reason to believe is thus connected with the conclusion should have more weight in determining our belief, than numerous properties that afford no evidence of such connection.

(3) In cases in which our inference is based upon the number of agreeing circumstances relative to the number of disagreeing circumstances, it is important that all points, both of agreement and difference, should be under view.

CHAPTER XV

FALLACIES

SECTION 37

FALLACIES INCIDENT TO REASONINGS UPON MATTERS OF FACT

IN this chapter we shall describe and explain those fallacies to which we are liable in the processes of reasoning which we have already explained. For convenience we divide fallacies into four groups, three of which correspond to the three special problems of science.

Accordingly, the first group includes the errors incident to the ascertainment of causal connection by observation and experiment. The second group includes errors of explanation by hypothesis; and the third group, the fallacies incident to calculation of chances and the statistical method. Into the last group we put the fallacies of generalization and false analogies.

Fallacies in Explanation by Observation and Experiment. — A part of the fallacies in our first group are commonly fallacies of observation; but, properly speak-

ing, observation is not fallacious; only inference, or discursive thinking, can be so characterized. Observation can be defective and otherwise faulty, and, consequently, be a psychological source of fallacies, as are our passions, our subjective biases of various sorts; but fallacy is committed only when we accept something as evidence for a belief which is either not evidence at all, or is inadequate evidence. Owing to defective observation, I may wrongly infer that there is a causal connection between two phenomena, *A* and *B*, just as passion or prejudice may cause me to give assent to a proposition for which there are no valid reasons; the fallacy in so doing consists in mistaking evidence.

This logical error must be distinguished from the psychological causes of it, which in these instances were wrong perception, prejudice, and passion. The common feature of all the fallacies of this first group is belief in causal connection in the absence of evidence, or upon insufficient evidence, the data not warranting the inference. Turning first to observation, as one of the fruitful sources of these fallacies, we note there are here two causes to which these fallacies are due: (1) oversight of material circumstances,'— non-observation; (2) confusion of perception with inference.

(1) An illustration of errors arising from oversight of material circumstances is afforded by advertisements of patent medicines, accounts of cures effected by them; only the successes being noted, the failures not observed. Another illustration is the belief that Friday is an unlucky day, only mishaps and fatalities

being observed, the fortunate incidents being overlooked.

What predisposes the mind to this fault in observation is the circumstance that coincidences impress us, and strongly so, if they are for any reason deeply interesting to us; that interest may be painful no less than pleasant; while coincidences on the other hand pass unnoticed. Carelessness, lack of interest, and consequently inattention to material circumstances are of course also causes of this mal-observation.

(2) The errors which arise from confusion of perception with experience are common enough, and almost unavoidable by untrained minds, without the corrections of experiment, or of careful comparison with the observations of others. Mr. Mill gives the following example of this form of wrong observation: "People fancied they saw the sun rise and set; and stars revolve in circles around the pole; and while they did so, they stubbornly refused to accept the Copernican theory." Inference here was mistaken for perception. People do not see the sun rise, etc.; they infer this from what they do see.

But, even were our observation faultless, and as complete as our faculties employed in the most careful manner could make it, even then the inference to causal connection would not be secure from fallacies. So great is the complexity of natural phenomena, so manifold and intricate the threads of connection between them, that no observation, however careful and however supplemented by experi-

ment, can disentangle this web and eliminate completely the non-causal circumstances in the case of any given phenomenon.

We have seen that the five methods of observation and experiment are the instruments by the use of which we attempt this resolution of phenomena and the discovery of causal connections; we have pointed out the limitations and defects of these methods; and we have seen what are the reasons why we can reach by them conclusions that are only probable, this probability in many cases being of a low degree. Hence, the fallacy to which we are exposed in relying upon these methods is that of overestimating the evidence they afford of causal connection.

Among these methods the one which most exposes us to this fallacy is the method of single agreement. The fallacy of *post hoc propter hoc* springs especially from the use of this method; and, inasmuch as it is this method that, from the nature of the case, is most commonly employed in reasoning upon political affairs, this fallacy is the almost universal sin of untrained reasoners, especially partisans and doctrinaires who dabble in sociological and economic matters. Examination of very many popular beliefs, those of which we are most tenacious, would show that they have no better ground than this customary conjunction in experience from which we infer a causal connection; they rest on the fallacy of *post hoc ergo propter hoc.*

Nor is the method of difference — the surest and most reliable of all the methods — exempt from this

liability to false inference. Suppose I am experimenting with the use of a particular kind of food in following this method; I select for comparison two instances, the instance in which I take the food and am made ill, and the instance in which I do not take the food and remain well. Now, could I be certain that my condition of health in the second instance was the same as in the first instance save in the one circumstance, that food was not taken in the one instance, and was taken in the other instance, then I should be justified in being certain of a causal connection between taking that food and being made ill.

But there is one unavoidable source of possible error in all such experiments: I must in this case compare two instances that are separated by some interval of time; and I can by no means be sure that in that interval a change has not taken place in my general condition, that was not a predisposing factor at least, in the illness which resulted from taking the food.

One other circumstance about these methods we are liable to overlook is their hypothetical and ideal character. As we have seen, they presuppose conditions to which we can only approximate in the actual situations which we most employ in them. Scientific investigators know these limitations of the methods they employ; and do not fall into the error of overestimating the evidence they afford; but minds not thus disciplined are apt to measure the strength of the evidence by the canons; and to overlook the fact that these canons

can be only partially complied with in our actual world.

Fallacies in Explanation by Hypothesis. — The fallacies of this group have their source in the processes of verification; and they arise either from an incomplete deduction and consequently incomplete comparison with facts, or from overlooking the difference between verification and complete proof. If the deduction from an hypothesis does not extend beyond the particular phenomenon it is constructed to explain, the evidence for the truth of that hypothesis is slight; oversight of this very limited test of the hypothesis leads to overestimation of the evidence in support of it. Even in case the deduction is extended to other phenomena, if those phenomena are of rather limited range and of simple character, we are liable to overestimate the evidence afforded by this fuller verification.

The disposition to be satisfied with incomplete verification has its source in our various subjective interests, propensions, and biases of mind; especially is this the case with social, economic, ethical, and religious hypotheses. If they accord with our habits, our inclinations, and favorite ways of viewing things, we are little disposed to look for facts that disprove them. The strength of our belief in them is not according to the objective reasons that exist, but according to their appeal to our subjective interests and appreciations. Most of these beliefs are extra-logical; and the fallacy we commit in them consists in oversight of the distinction between logical grounds and psychological motives or causes.

The other fallacy in verification has its source in mistaking complete verification for complete proof. It is substantially the fallacy of affirming the consequent in the hypothetical syllogism. To draw the conclusion that an hypothesis is certainly true because the given phenomena are just what they would be if this hypothesis were true, overlooks the possibility of what Mill calls plurality of causes.

There is one more error, somewhat the converse of this fallacy, which arises from overlooking the difference between disproof and incomplete verification. Objectors to a given hypothesis are prone to this fallacy of inferring that an hypothesis is false if it fails to explain all the phenomena. Critics of Darwin drew such an inference from his frank admission that there were outstanding facts which his theory did not satisfactorily explain; and some of these objectors were confident that Darwin had logically abandoned his hypothesis, when he admitted, that if there was a single fact that could not be harmonized with his hypothesis, he must abandon it. The unexplained facts were at once pointed out, with the triumphant inference that Darwin's theory was overthrown by his own admission. Now, the fallacy which these logical refuters of Darwin committed, is the fallacy of confounding facts not explained by an hypothesis with facts that contradict that hypothesis.

Fallacies Incident to the Calculation of Chances and the Method of Statistics. — The theory of chance, we have seen, assumes that the causes which determine

the particular event are so adjusted to each other in their influence, that, in a sufficiently long run, as many events of one particular sort will occur as of any other sort of possible events; any one side of the die will come up as many times as any one of the other five sides. Consequently, when, in a limited succession of instances, the same event occurs a disproportionate number of times, the inference seems to be justified that some particular cause is operative. In some cases it has been shown that this inference is incorrect; and the error is a consequence of overlooking the tendency of such phenomena to a sort of rhythmic recurrence of the same succession; that is, a run of events of the same character — six-spots — is followed by a run of those of different character, — say aces, — and a return of the run to sixes, etc.

It should not, however, be maintained that this fallacy of inferring a special cause is committed in all cases where a like excess of actual over theoretical coincidences occurs. In the phenomena of telepathy, or alleged thought transference, it is very confidently maintained that the coincidences are so much and so persistently in excess of mere chance coincidences, that no other conclusion is reasonable but the existence of a particular cause. It does not, however, come within the province of logic to determine the truth or error of such inferences. If they should be proved to be erroneous, the fallacy would be the result of mistaking recurring non-causal coincidences for causally connected ones; and the source of this error would be

an oversight of the tendency of such phenomena to form habits, — of the same sort as are shown in dice throwing, selecting cards, balls from a box, etc.

The method of statistics is liable to two special fallacies: one of these is that of inferring a causal determination of a particular phenomenon from uniformity which holds true of a group to which this particular phenomenon belongs; thus, from the fact that in a certain city the rate of mortality is $\frac{1}{100}$, it is inferred that A, who lives in that city, has ninety-nine chances of living against one of dying in a specified period of time.

This fallacy is akin to the fallacy of assuming that whatever can be affirmed of a class can also be affirmed of every individual of that class; but the exact cause of this fallacy is an erroneous conception of the statistical method, and what that method assumes; and the fallacy really consists in confounding the statistical method with the calculation of chances. The other fallacious use sometimes made of statistics is the attempt to prove by means of them that human actions have natural causes, as do all phenomena in nature; in other words, the attempt is made to establish a doctrine of determinism by the aid of this method. Human actions, so runs the argument, cannot be free, because it is ascertained that in any specified group of persons, and for a given period of time, a relatively constant number of actions of a particular sort, say murder, are performed; there must be some constantly acting cause, and such a cause is incompatible with free will.

Now, such an argument involves a twofold misconception: a misconception of the method of statistics, and a misconception of the nature of free action and the conditions of it. The latter misconception is a metaphysical one, and the exposure of it does not fall to the logician. The first error overlooks the fact that the method of statistics does not consider individuals as individuals in the averages and constant ratios it establishes. It is entirely erroneous to infer that, since causes determine this constant ratio, they do so by determining the actions of all the individuals who composed the group. There is room in such aggregates as the method of statistics contemplates, for all the individual freedom of choice that the stoutest champion of this doctrine need contend for.

Fallacies in Generalization and in Analogy. — The fallacy to which inductive generalization is prone is that of generalizing from too narrow a datum. Instances of such unjustified inferences abound in popular beliefs. Critical examination would surprise most people by the discovery that very many of the beliefs they think are securely based, have really no other foundation than an experience of narrow range. In many cases it is an experience of few instances on which the broad generalization rests, especially if those instances are such as excite or appeal strongly to our predilections and our prejudices, our likes and our aversions.

The absence of contradictory instances greatly helps this tendency to hasty generalization. But perhaps the

strongest factor in inclining our minds to premature generalization is a disposition, a structural principle of our minds, and rational in itself, — the disposition to look for uniformity, the prejudice in favor of order, the propensity to see in all cases not yet under observation the same nature that we find in those we know. We want uniformity; we want the future to be like the present; we are impatient of delay; we dislike to wait for contradictions of what seems to be the uniformity we have already begun to accept, and especially if it be a uniformity that is congenial to us.

A practical interest asserts itself in these generalizations and tends to make them hasty; it is the need for action, for adjustment to coming situations, especially to the behavior of our social fellows. If what I have found to be true of some men I can *now* believe to be true of men I have yet to know personally, I can the more successfully plan my future actions in reference to them; knowing what I would have to expect from them in any situation, I can prepare to meet that situation. Thus we hurry our generalizations, because we have practical interests that we think will not wait the slow testing of experience.

Thus, a variety of motives and interests, partly emotional, partly intellectual, and partly practical, conspires to this error of generalization from insufficient data, conspires to produce this illusion of the logical faculty, leading it to overestimate the evidence for its conclusion.

Our exposition of analogical reasoning has made it quite obvious that the one fallacy to which it is exposed consists in counting resemblances or differences instead of weighing them. The unpracticed reasoner is prone to suppose the inference is necessarily strong in proportion to the excess of the points of likeness over the points of difference; and he is liable to overlook the importance either of the resembling properties or of the properties that present differences. Reasoning upon social and political matters is a fruitful field for false analogies. No argument is more specious, and no argument so readily captivates the imagination and is so successful with the ordinary mind.

APPENDIX

PRACTICAL EXERCISES AND QUESTIONS

CONCEPTS, NAMES, TERMS

1. Explain the following distinctions: simple, complex, universal, singular, collective, concrete, abstract, positive, negative, absolute, relative.

2. In the following, which concepts are universal, which are singular, and which are collective? man, army, stone, science, congress, Edward VII, charity, union, Venus, the Pope, courage, the year 1905.

3. State whether the following concepts in the instances given are concrete or abstract: quality, mercy, justice, meekness, truth, righteousness, blue, humanity, nation, nationality, equality.

(1) The quality of mercy is not strained.
(2) Mercy rejoiceth against judgment.
(3) Meekness is a virtue in a strong man.
(4) Truth crushed to earth shall rise again.
(5) Righteousness exalteth a nation.
(6) The sky is blue.
(7) Humanity is destined to finer development.
(8) He is of Greek nationality.

4. Give the logical characteristics of the following (by logical characteristics are meant the distinctions, complex, simple, universal, singular, collective, etc.): —

man	charity	parliament
conscience	sweetness	king
wood	Pole	father
virtue	the sun	genius
peer	feeling	army
happiness	million	John Smith

5. How do you explain extension, intension, connotation, and denotation of a name or concept?

6. Arrange the following with reference to extension and intension, placing them in the order of diminishing extension: man, animal, ruler, living being, emperor, king of England, quadruped, horse, racing horse, body, star, Sirius, heavenly body.

7. Explain division, classification, and definition, and the relation between division and definition.

8. Divide the following, carrying the division three steps in each instance: trees, books, buildings, government, sciences, metals.

9. Explain genus, species, difference, property, and accident, and give an example of each.

10. What distinction can be made between accident and property, and between proximate species and lowest species?

11. Give definitions of the following: circle, history, quadruped, poetry, money, government, the North Pole.

12. What are the faults in the following definitions: —

(1) Man is a two-legged animal without feathers.

(2) A triangle is a plane figure having three sides, three angles, and the sum of the angles is equal to two right angles.

(3) A king is an hereditary ruler of a kingdom having an extensive area and a dense population.

(4) Vice is the opposite of virtue.

(5) A gentleman is a man having no visible means of subsistence.

(7) Man is a self-knowing animal.

13. Examine the following divisions and state why they are faulty: —

(1) Rectilinear figures are divided into triangles, parallelograms, rectangles, and polygons of more than four sides.

(2) Wars are divided into civil and destructive.

(3) Allegiance is either natural and perpetual or local and temporary.

JUDGMENTS AND PROPOSITIONS

1. Distinguish a judgment from a grammatical sentence.

2. In what different ways can a judgment be expressed?

3. Explain the hypothetical and the disjunctive judgments.

4. Analyze the following sentences. (The logical analysis of a sentence is reduction of it when necessary to the proposition or propositions it contains, the propositions being made as simple as possible so as clearly to

express the relation between the subject and predicate terms. Thus, the sentence, "They have rights who dare maintain them," analyzed gives, "Those who dare maintain their rights are those who have rights"; and the sentence, "Where there's a will there's a way," gives, "The situations in which there is a will are the situations in which there is a way," or "To have a will is to find a way." The two relations between subject and predicate terms to which propositions can be reduced are the relation of attribute to subject or of class to class; and in the analysis of sentences it is best to reduce them to propositions which assert one or the other of these relations.)

(1) Then to side with truth 'tis noble, when we share her wretched crust.
(2) Truth crushed to earth shall rise again.
(3) Honor and shame from no condition rise.
(4) Some murmur when their sky is clear.
(5) Castles of the great are jails.
(6) Stone walls do not a prison make, nor iron bars a cage.
(7) 'Tis equal peril to go or to remain.
(8) He who is capable of making a pun is capable of picking a pocket.
(9) New occasions teach new duties.
(10) They must ever up and onward who would keep abreast of truth.
(11) All's well that ends well.
(12) All cannot receive this saying.
(13) A friend should bear a friend's infirmities.

(14) The path of glory leads but to the grave.
(15) Few shall part where many meet.
(16) One ruddy drop of manly blood the surging sea outweighs.
(17) A little consideration of what takes place around us every day would show us that a higher law than our wills regulates events.
(18) There is less intention in history than we ascribe to it.
(19) What we do not call education is more precious than what we call so.
(20) Laurel crowns cleave to deserts, and power to him who power exerts.

5. Distinguish the kinds of judgments and the kinds of propositions in the following: —

(1) Whatever is, is right.
(2) Planets revolve from east to west.
(3) If a body is heated, it expands.
(4) All exercises are not so easy as they look.
(5) Honor lost, all is lost.
(6) Where there's a will there's a way.
(7) Plane figures are either rectilinear or curvilinear.
(8) What is not an element is not a metal.
(9) Life's not all beer and skittles.
(10) How vain that second life in other's breath!
(11) We must either control our passions or be their slaves.
(12) He can't be wrong whose life is in the right.

(13) Only the actions of the just smell sweet, and blossom in the dust.

(14) Good men sometimes do wrong.

(15) Some men are always in the wrong.

(16) Few shall part where many meet.

(17) Only honorable actions deserve praise.

(18) Fame is no plant that grows on mortal soil.

(19) Some of those present were all who engaged in the sport.

(20) Only a few did not escape.

6. Explain the negative judgment, and illustrate by an example the positive value of negation.

7. What do negative propositions assert?

8. What is the quality of the following propositions:

(1) Few and short were the prayers we said.

(2) Few shall part where many meet.

(3) What is not an element is not metal.

(4) Only members vote.

9. Explain quantification. Show how it applies to each term in the proposition.

10. By using the symbols *A*, *E*, *I*, and *O*, indicate the quantity and quality of the propositions given under question 5.

11. Quantify the following propositions.

(In quantifying reduce each proposition to one of the formulæ explained under the topic Quantification. Also make use of the circles to illustrate quantification.)

(1) *A* and *B* are always together.

(2) Some murmur when their sky is clear.

(3) All lawyers are not knaves.

(4) All the perfumes of Arabia will not sweeten this little hand.
(5) None know where the shoe pinches but the wearer.
(6) He is gentle that doeth gentle deeds.
(7) Most men prefer their own interests.
(8) Not every one who saith to me Lord, Lord, shall enter the kingdom.
(9) To be great is to be misunderstood.
(10) Only a few were saved.
(11) Some of the voters were the only ones who voted.
(12) A fool at forty is a fool indeed.
(13) No one is always happy.
(14) A stitch in time saves nine.
(15) Not many wise men are called.
(16) Few are chosen.
(17) We spoke not a word of sorrow.
(18) There is none good but one.
(19) Uneasy lies the head that wears the crown.
(20) The North Pole is not yet reached.

IMMEDIATE INFERENCE

1. Explain and illustrate equipollence and opposition.

2. Name the different logical relations under equipollence and under opposition.

3. Arrange the following propositions in pairs so that a logical relation shall be shown in each pair of propositions, and state what this relation is: —

(1) All material substances possess gravity.
(2) No material substances do not possess gravity.
(3) What does not possess gravity is not a material substance.
(4) No material substances possess gravity.
(5) Some material substances do not possess gravity.
(6) Some things which possess gravity are material substances.
(7) Some things not possessing gravity are material substances.
(8) No substances possessing gravity are material substances.

4. Obvert the following propositions: —
(1) All metals are useful.
(2) All organic substances contain carbon.
(2) All mammalians are vertebrates.
(4) Whatever is necessary exists.
(5) No men are always happy.
(6) Nothing great is easy.
(7) Some mistakes are culpable.
(8) Few escape misfortune.
(9) Some mistakes are not culpable.
(10) Only members vote.

5. Convert the following: —
(1) A stitch in time saves nine.
(2) Few shall part where many meet.
(3) A man's a man for a' that and a' that.
(4) Uneasy lies the head that wears the crown.
(5) Every little makes a mickle.

(6) Few men are perfectly content.
(7) No men are absolutely bad.
(8) Where there's a will, there's a way.
(9) All's well that ends well.
(10) *A* and *B* are always together.

6. State the contrapositive to the following: —
(1) All living tissue is organic.
(2) No man loves dishonor.
(3) Some promises are better broken than kept.
(4) Knowledge is power.
(5) Some mistakes are not culpable.

7. Contradict and give the contrary to the following: —
(1) Whatever is, is right.
(2) All the world is gone after him.
(3) No wrong can remain unrighted.
(4) Some men are always in the wrong.
(5) No man is a hero to his valet.

8. Give every opposition possible to the following: —
(1) Life is not all beer and skittles.
(2) Some just acts are inexpedient.
(3) All's well that ends well.
(4) The longest road has a turn.
(5) Only the brave deserve the fair.
(6) Whatever is, is right.
(7) Some good actions are not rewarded.
(8) None but brave men dare always do right.
(9) All roads lead to Rome.
(10) Men of fair promises are often not to be trusted.

9. From each of the following propositions, as a premise, pass to as many propositions as are logically related to the given proposition: —

(1) Perfect happiness is impossible.
(2) Few are acquainted with themselves.
(3) Talents are often misused.
(4) No knowledge is useless.
(5) He jests at scars who never felt a wound.
(6) Familiarity breeds contempt.
(7) All's well that ends well.
(8) Some stars are not seen.
(9) Nothing is worth doing at all that is not worth doing well.
(10) Every little makes a mickle.

MEDIATE INFERENCE. THE SYLLOGISM

1. Construct a categorical, a hypothetical, and a disjunctive syllogism respectively.

2. Explain major, minor, and middle terms.

3. Define figures of the syllogism and construct a syllogism in each of them.

4. Explain moods and the maximum number of them.

5. In the categorical syllogism, prove the following rules: —

(1) One premise at least must be universal.
(2) One premise at least must be affirmative.
(3) In Fig. I the minor must be affirmative.
(4) In Fig. II one premise must be negative.
(5) In Fig. I the major premise must be universal.

6. Construct a dilemma, a sorites, and an epichirema, and distinguish between constructive and destructive, and between complex and simple dilemmas, also between the Aristotelian and the Goclenian sorites.

7. Expand the following enthymemes into complete syllogisms: —

(1) Blessed are the meek, for they shall inherit the earth.

(2) Some pleasures are not praiseworthy, hence some pleasures are not honorable.

(3) A nation may depose a bad king; for it has a right to good government.

(4) Law is an abridgment of liberty, and consequently of happiness.

(5) If he did not steal the goods, why did he conceal them as no thief ever fails to do?

(6) He cannot be a gentleman, for no gentleman would do such a thing.

(7) A body cannot move; for to do so it must move where it is or where it is not and it can do neither of those things.

(8) If it is fated that you are to recover you will do so whether you call in a doctor or not, and if it is fated that you will not recover, it is useless to call in a doctor; consequently, it is useless to call in a doctor.

(9) Our ideas reach no farther than our experience; we have no experience of obvious attributes.

(10) I infer that some stupid persons must have passed in the last examination.

(11) Discontent is an essential condition of progress, — but discontent means sorrow.

(12) That which causes a balance of good is right; therefore, persecution may sometimes be right.

(13) How can one maintain that the insane should never be punished who maintains that they should always be benefited?

(14) No man should fear death; for it is according to nature.

(15) All human things are subject to decay; and when fate summons, monarchs must obey.

(16) He is free who lives as he wishes; the bad, therefore, are not free.

(17) Blessed are the peacemakers, for they shall be called the children of God.

(18) There can be no religion without infallibility; for no religion is possible without a visible church, no church without government, no government without sovereignty, and no sovereignty without infallibility.

(19) All love happiness; all love life.

8. What premises have the following propositions: —

(1) He is not a wise man.

(2) Knavery and folly sometimes go together.

(3) Some victories are won by accident.

(4) He must die.

(5) A college education is not always necessary to success.

(6) The meek must be blessed.

9. Draw conclusions from the following premises where a conclusion is admissible, and give reasons why some of the premises do not give conclusions.

(1) No birds are biparous; all feathered animals are birds.

(2) Sodium is a metal; sodium is a very dense substance.

(3) Violations of law should be punished; lying is not a violation of law.

(4) Most men prefer their own interests; A is a man.

(5) All men are mortal; no men are perfect.

(6) Scarlet fever patients have high temperature; X has a high temperature; X has not a high temperature.

(7) Only express trains do not stop at this station; the last train was an express train.

(8) If it rains, he will not come; he did not come.

(9) If it rains, he will not come; it did not rain.

(10) If A is B, C is D; and if E is F, G is H; C is not D and G is not H.

LOGICAL FALLACIES

1. Explain the fallacies in the following, giving the technical names for them: —

(1) All who think this man innocent think he should not be punished; you think he

should not be punished; therefore you must think he is innocent.

(2) Whatever is vicious should be punished; intemperance is not vicious and therefore should not be punished.

(3) He must have stolen the goods, for he concealed them, which every thief does.

(4) Everything permitted by law is morally right; and therefore whatever is morally right is permitted by law.

(5) Express trains only do not stop at this station; and as the last train did stop, I infer it was not an express train.

(6) Over credulous persons ought never to be believed: and since some ancient historians are untrustworthy, they must have been over credulous.

(7) Nearly all the satellites revolve from west to east; the moon therefore must revolve from west to east.

(8) Had Pitt carried out the doctrine of free frade, he would have been a great statesman; but he did not carry out that doctrine; therefore he was not a great statesman.

MATERIAL FALLACIES

1. Describe the following fallacies: accident fallacy, fallacy of composition, and the converse fallacy of division.

2. How distinguish between the fallacy of ambiguity and accident fallacy?

3. What is fallacy *a dicto simpliciter ad dictum secundum quid?* What is the converse fallacy and how are these fallacies related to the accident fallacy?

4. Distinguish between *petitio principii* and *ignoratio elenchi.*

5. In what two ways is *petitio principii* committed, and what forms of *ignoratio elenchi* are there?

6. Describe and name the fallacies in the following arguments: —

(1) Slavery is a natural institution, and therefore it ought not to be abolished.

(2) We know that God exists, because the Bible tells us so; and we know that whatever the Bible affirms is true, because it is of divine origin.

(3) Nations are justified in revolting when badly governed, because every nation has a right to good government.

(4) Some holder of a ticket is sure to draw the prize; and, as I am a ticket holder, I am sure to draw the prize.

(5) What fallacy did Columbus commit, when he made the egg stand on end by breaking in the end of the egg?

(6) What fallacy was the humorist afraid of when he said he would not accept a demonstration in mathematics, until he knew what use was to be made of it?

(7) Improbable events happen every day; now, what happens every day is a probable event; therefore, improbable events are probable events.

(8) A miracle is incredible because it contradicts the laws of nature.

(9) Every hen comes from an egg, every egg comes from a hen; therefore, every egg comes from an egg.

(10) The Germans are beer drinkers; Hans, being a German, must also be a beer drinker.

(11) What we eat grew in the fields; loaves of bread are what we eat; therefore, loaves of bread grew in the fields.

(12) Wine is a stimulant; therefore, in every case where a stimulant is harmful, wine is harmful.

(13) Gold and silver are the wealth of a country; consequently the diminution of gold and silver by exportation must be the diminution of the wealth of a country.

(14) If I am to pass this examination, I shall pass it whether I answer correctly or not; if I am not to pass it, I shall fail whether I answer correctly or not; therefore, it is of no consequence how I answer the questions.

(15) All the trees in the park make a dense shade; this oak is a tree in the park, and consequently it makes a dense shade.

(16) Whoever intentionally kills another should suffer death; a soldier should therefore suffer death, since he intentionally kills.

(17) Every rule has exceptions; this statement is a rule, and therefore has exceptions; therefore, there are some rules that have no exceptions.

(18) Repentance is a good quality; wicked men abound in repentance; and therefore they abound in what is good.

(19) Meat and drink are the necessaries of life; the revenues of the king were spent on meat and drink, and consequently they were spent on the necessaries of life.

(20) We charged him (King Charles the Second) with having broken his coronation oath, and we are told that he kept his marriage vows; we accuse him of having given up his people to the merciless infliction of the most hot-headed and hard-hearted of prelates, and the defence is that he took his little son on his knee and kissed him; we censure him for having violated the articles of the Petition of Rights, after having for a good and valuable consideration promised to observe them, and we are informed that he was accustomed to hear prayers at six o'clock in the morning.

What fallacy does Macaulay refer to in this passage?

(21) This must be a bad measure, because it is supported by bad men.

(22) For those who are bent on cultivating their minds by diligent study, the incitement of academic honors is unnecessary; and it is ineffectual for the idle, and such as are indifferent to mental improvement; therefore, the incitement of academic honors should be abolished.

(23) Logic as it was cultivated by the Schoolmen was a useless study; therefore, logic as it is cultivated to-day is a fruitless study.

(24) Protective laws should be abolished; for they are injurious if they produce scarcity, and they are useless if they do not.

(25) What is the good of all your teaching, for every day we hear of forgeries, which would never have been committed by these persons, had they not learned to read and write?

(26) Does a grain of millet, when dropped on the floor, make a sound? No. Does a bushel of millet under these same circumstances make a sound? Yes. Is there not a determinate proportion between the bushel and the grain? There is. There must, therefore, be the same proportion between the sonorousness of the two. If one grain be not sonorous, neither can ten thousand grains be so.

(27) He that can swim need not despair to fly; for to swim is to fly in a grosser fluid, and to fly is to swim in a subtler fluid.

(28) The more correct the logic, the more certainly will the conclusion be wrong, if the premises are false; therefore, where the premises are wholly uncertain the best logician is the least safe guide.

(29) The two propositions, Aristotle is living and Aristotle is dead, are both intelligible propositions; they are both of them true or both of them false, because all intelligible propositions must be either true or false.

(30) Every incident in the narration is probable; hence the narrative is probable.

(31) The end of a thing is its perfection; death is the end of life, and therefore death is the perfection of life.

(32) It is enough to reply to your argument, when I remind you that of all men you should be last to advocate this doctrine.

(33) Every law is either useless or it occasions hurt to some person; now, a law that is useless ought to be abolished, and so ought a law that occasions harm; therefore, every law ought to be abolished.

(34) All the plays of Shakespeare cannot be read in a day; "Hamlet" is a play of Shakespeare, and consequently it cannot be read in a day.

(35) You are not what I am; I am a man; therefore, you are not a man.

(36) Theft is a crime; theft was encouraged by the laws of Sparta; therefore, the laws of Sparta encouraged crime.

(37) The Greeks produced masterpieces of art; the Spartans were Greeks, and therefore they produced masterpieces of art.

(38) He is the greatest lover of any one who seeks that person's greatest good; a virtuous man seeks the greatest good for himself; therefore the virtuous man loves himself most.

(39) The student of history is compelled to admit the law of progress, for he finds that society has never stood still.

EXAMPLES OF THE METHODS OF OBSERVATION AND EXPERIMENT IN ASCERTAINING CAUSAL CONNECTION

1. Baron Liebeg investigated the actions of certain metallic poisons. His problem was to ascertain the property common to arsenious acid, salts of lead, bismuth, copper, and mercury on which their destructive action was dependent. He ascertained the following facts: —

(1) When solutions of these substances are placed in sufficiently close contact with many animal products, albumen, milk, muscular fiber, and animal membranes, the acid or salt leaves the water in which it was dis-

solved, and enters into combination with the animal substances; which substance, after being acted upon, loses its tendency to spontaneous decomposition or putrefaction.

(2) In all cases where death has been produced by these poisons, those parts of the body with which these substances have been brought into contact do not afterwards putrefy.

(3) When too small a quantity of poison has been used to destroy life, eschars are produced, superficial portions of the tissues are destroyed.

(4) Many insoluble basic salts are known not to be poisonous; when these, however, are brought into contact with tissues, they do not combine with them so as to arrest the process of decomposition.

(5) Antidotes to these poisons are known to combine with them so as to prevent them from acting upon the tissues, by forming insoluble compounds.

The conclusion reached by Baron Liebeg was that the proximate cause of death from the action of these poisons is the conversion of animal tissues into a chemical compound, held together by so powerful a force as to resist the action of the ordinary causes of decomposition on which the continuance of life depends.

By what method was this conclusion reached?

2. Dr. Wells' investigation upon the cause of nocturnal dew: —

The following facts were ascertained: —

(1) Whenever an object contracts dew it is colder than the air.

(2) No dew is produced on the surface of polished metals, but dew is produced on the surface of glass.

(3) Polished substances which conduct heat least were found to be most conspicuously bedewed, while those which conduct it will resist dew.

(4) Substances which part with their heat most readily by radiation contract dew most abundantly.

(5) All the instances in which much dew is deposited agree in this circumstance only, that they either radiate heat rapidly, or conduct it slowly. Bodies that are bedewed are those that lose heat from the surface faster than it is restored from within.

(6) Dew is never deposited copiously in situations much screened from the open sky and not at all in a cloudy night.

Which of the methods were followed in ascertaining these facts, and in reaching the conclusion in 5?

3. What method was followed by Arfwedson in his discovery of lithia by noting an excess of weight in the sulphate produced from a small portion of what he considered as magnesium present in a mineral he had analyzed?

4. Jevons observed that economic crises have occurred at regular intervals of about ten years; this ten years' periodicity, moreover, seems to correspond to a similar periodicity of bad harvests; and the causes of this seem to be a decennial periodicity of spots on the sun.

5. In a simple fracture of the ribs if the lung be punctured by a fragment, the blood effused into the pleural cavity, although freely mixed with air, undergoes no decomposition. That is not the case if air enter directly through a wound in the chest. This difference in result must be causally connected with special circumstances — viz. passage of air through tissues in the lungs.

What method is illustrated in these observations?

6. If the lung be emptied as perfectly as possible, and a handful of cotton wool be placed against the mouth and nostrils, and you inhale through it, it will be found on expiring this air through a glass tube that its freedom from floating matter is manifest.

What two circumstances are shown to be causally connected in this experiment and by what method?

7. The following experiments, it is maintained, prove that the feeling of effort is of peripheral rather than central origin.

(1) Hold the finger as if to pull a trigger; think vigorously of bending it but do not bend it; an unmistakable feeling of effort results. Note in repeating this experiment, that the breath is involuntarily held, and that there are also other muscle contractions.

(2) Now, repeat the experiment, and breathe regularly at the same time and avoid other muscle contractions, and note that no feeling of effort is the result. What method is followed in these experiments?

8. Darwin asserted that cross fertilization of the flower of the common broom by bees is causally connected with a curious mechanism in these flowers. The following circumstances were noted by him: —

(1) "When a bee alights on the petals of a young flower it is slightly opened, and short stamens spring out, which rub their pollen against the abdomen of the bee. If a rather older flower is visited for the first time (or if the bee exerts great force on a younger flower), the keel opens along its whole length, and the longer as well as the shorter stamens, together with the much elongated curved pistil, spring forth with violence. The flattened spoonlike extremity of the pistil rests for a time on the back of the bee, and leaves on it the load of pollen with which it is charged. As soon as the bee flies away, the pistil instantly curls round, so that the stigmatic surface is now upturned and occupies a position in which it would be rubbed against the abdomen of another bee visiting the same flower. Thus, when the pistil first escapes from the keel, the stigma is rubbed against the back of

the bee, dusted with pollen from the shorter stamens, which is often shed a day or two before that from the longer stamens. If the visits of bees are prevented, and if the flowers are not dashed by the wind against any object, the keel never opens, so that the stamens and pistil remain inclosed. Plants thus protected yield very few pods in comparison with those produced by neighboring uncovered bushes, and sometimes none at all." Quoted from Darwin in Hibbens's "Inductive Logic," pp. 316–317.

9. Kenelm Digby's treatment of wounds was to apply an ointment, not to the wound itself, but to the sword that had inflicted it, to dress this carefully at regular intervals, and, in the meantime having bound up the wound, to leave it alone for seven days. It was observed that many cures followed upon this treatment.

What fallacies does this incident illustrate?

10. What fallacy underlies the saying "Fortune favors fools"?

11. By what fallacious methods is the success of patent medicines largely promoted?

12. To what were the following beliefs chiefly owing?

A body ten times as heavy as another falls ten times as fast. Objects immersed in water are always magnified. The magnet exerts an irresistible force. Crystals are always found associated with ice.

13. A belief was current in Adam Smith's time that prodigality encourages industry and parsimony discourages it. Observation seemed to justify this belief; those who spent lavishly gave great employment to labor. Those who were not thus lavish did not appear to do so.

What faults of observation were the cause of this erroneous belief?

14. What mal-observation was there in the objection to free trade, that the purchase of British silk encourages British industry, the purchase of Lyons silk encourages only French industry?

15. What error lies in the following beliefs? Whatsoever has never been will never be. Women as a class are not equal to men. Society cannot prosper without slavery. Philosophers are impractical men.

16. What fallacy can you charge against the following arguments?

> "As there could be in natural bodies no motion of anything unless there were some which moveth all things, and continueth immovable; even so in politic societies there must be some unpunishable, or else no man shall suffer punishment."
>
> "It would be admitted that a great and permanent diminution in the quantity of some useful commodity, such as corn, or coal, or iron throughout the world, would be a serious and lasting loss; and again, that if the fields and coal mines yielded regularly double quantities,

with the same labor, we should be so much the richer; hence it might be inferred, that if the quantity of gold and silver in the world were diminished one half, or were doubled, like results would follow; the utility of these metals for the purpose of coin being very great."

ARGUMENTATION AND DEBATE

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12mo Cloth xviii + 361 pages $1.10 net

In this work the peculiar difficulties which stand in the way of making a *practical text-book* for use in teaching argumentation and debate have been overcome. The authors have succeeded in producing a book which is not only practical and teachable, but which has the still rarer quality of being *easily understood.* The treatment of the topics presented — the proposition, the issues, preliminary reading, evidence, kinds of arguments, fallacies, brief-drawing, the principles of presentation, refutation, and debate — is lucid and interesting as well as highly profitable.

The discussion of the *Issues* is built around the vital statement that, "in arguing, there are always certain ideas or matters of fact, upon the establishment of which depends the establishment of the proposition." It is shown that there is no sure way of guarding against irrelevant discussion, except by clear understanding and concise statement of the issues. *The method of finding the issues* is fully explained and is also illustrated by quotations from the speeches of great debaters.

The discussion of *Evidence* rests on the broad legal basis. Under this head are included a careful analysis of the kinds of evidence, and a discriminating statement of the relative value of evidence, while the various tests of evidence are suggested for the consideration of the student. At the close of the chapter on Evidence will be found an illustration of the method employed, which makes the book *eminently teachable.* The whole chapter is summarized in such a way that, not only is the student greatly assisted in fixing in his memory the various topics, but the instructor likewise finds great help in *effective quizzing.*

The difference between Evidence and Arguments is made clear in the chapter on *Kinds of Arguments*. It is strongly maintained that the different kinds of Arguments may all be best explained by direct reference to the causal connection between that which is known and that which is to be inferred. In every case the discussion is strengthened by well-chosen illustrations from standard sources.

Perhaps the *most distinctive chapter* in the book is that on *Brief-drawing*. This subject is so presented that even the beginner, after careful study of the chapter, is ready to commence the work for himself. The plan of the chapter is as follows: (*a*) A subject in the form of a proposition is selected; (*b*) a rough outline is made; (*c*) this rough outline is modified step by step; (*d*) there are discussions of each change and the formulation of a rule of brief-drawing, and finally (*e*) the completed brief is given as a whole. At the end of the chapter are provided all the rules that have been enunciated for the drawing of briefs. This method of presenting this most difficult part of the work is, it is believed, vastly superior to any other method that has been tried, in that it gives the student *one* model, drawn on approved lines and presented clearly, thus guarding against what too often becomes a discouraging jumble.

For *apt Illustrations*, that actually illuminate the processes under discussion, the great preachers, platform speakers, and forensic orators of ancient and modern times, have been so laid under contribution, that the result is practically an anthology of argumentation and debate.

In the work under consideration it is fully realized that something more is needed for Debate than for written discussion, and so the Part on *Debate* contains added suggestions of a highly practical nature.

Teachers will find that the *careful paragraphing, lettering, and summarizing*, which have been done with great exactness throughout the entire book, help in no small measure to make the subject of Argumentation and Debate *eminently practical for class use*. The Appendixes contain suggestions of additional exercises that have proved helpful in the teaching of the subject.

THE MACMILLAN COMPANY

64–66 FIFTH AVENUE, NEW YORK

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